

*Good evening!
Thank you for coming.*



*This evening we will provide you with
information on how we teach
Mathematics at Moorlands and suggest
ways that you can help your child at
home using similar methods.*



Maths at Moorlands.

*A focus on Concrete, Pictorial and
Abstract approaches. (CPA)*

Karen Chapman, Sophie Crowther and
Emma Bancroft

Maths Leaders Moorlands Primary School

With thanks to Kelsey Brown, White Rose Maths Hub

Agenda



- *The theory behind the importance of CPA*
- *Using manipulatives to introduce the basics of a new concept and ways to replicate this at home.*
- *Transitioning between concrete, pictorial and abstract.*



Concrete, Pictorial and Abstract Methods

Importance of CPA



In his research on the cognitive development of children (1966), Jerome Bruner proposed three ways of working to aid development:

- Enactive representation (using 'concrete' objects)
- Iconic representation (drawing images / pictures)
- Symbolic representation (abstract numbers)

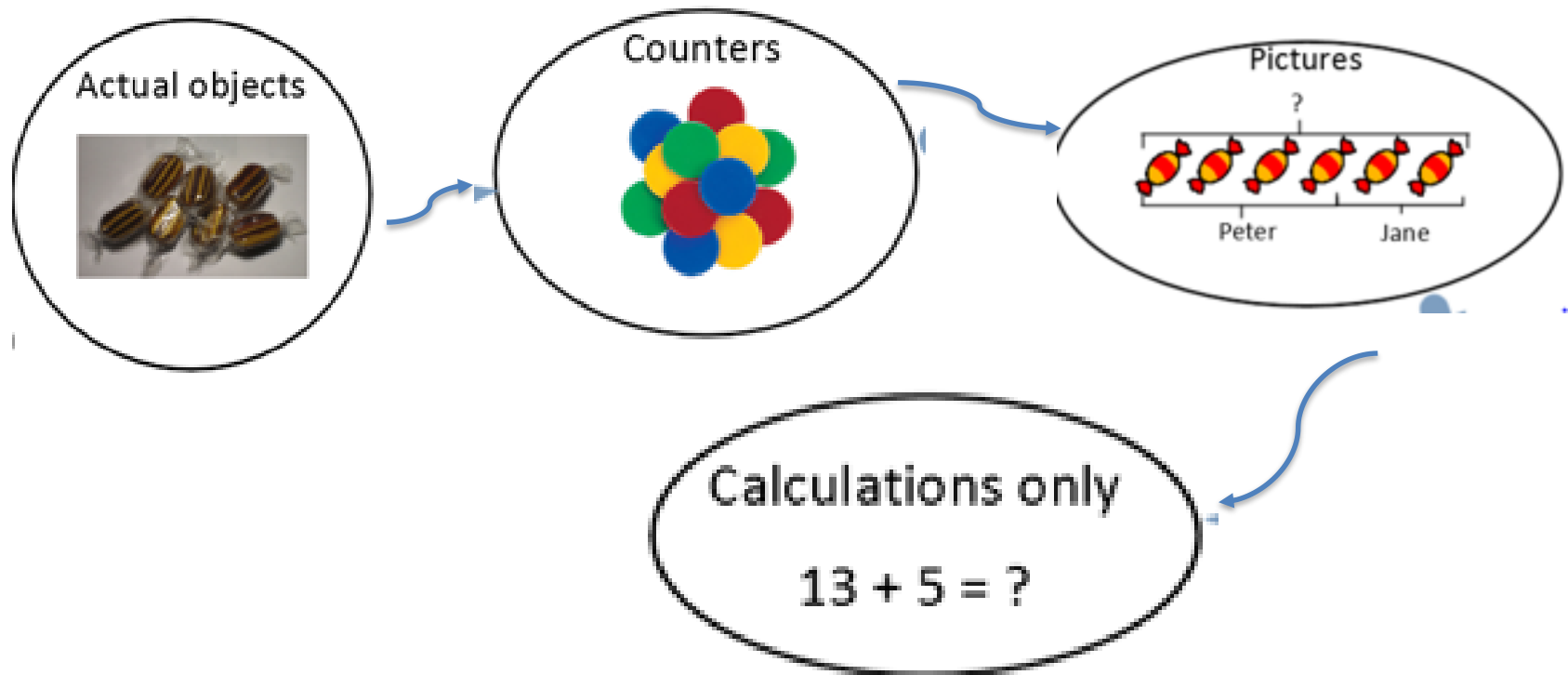
“If we do not use concrete manipulations, then we can not understand mathematics. If we only use concrete manipulations, then we are not doing mathematics.”

Gu (2015)



Concrete, Pictorial, Abstract

Children should work at the stage they need until ready to move on. Within a class children can be working on the same calculation but accessing it in different ways.



Using CPA methods



Today we aim to give you a quick insight into methods used at school and how you could adapt those to work at home, with a focus on:

- *Place value*
- *Addition and Subtraction*
- *Multiplication and Division*
- *Fractions and Problem Solving*



Place Value

*(understanding the value of
each digit and it's place in
the number system)*

Place value

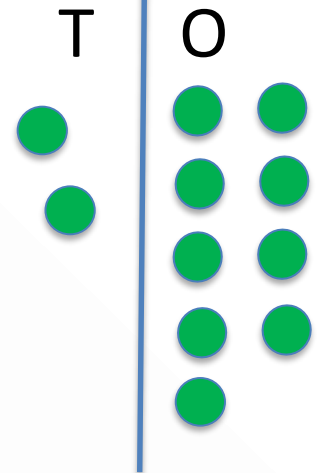


How many ways can you show 29?

Twenty nine

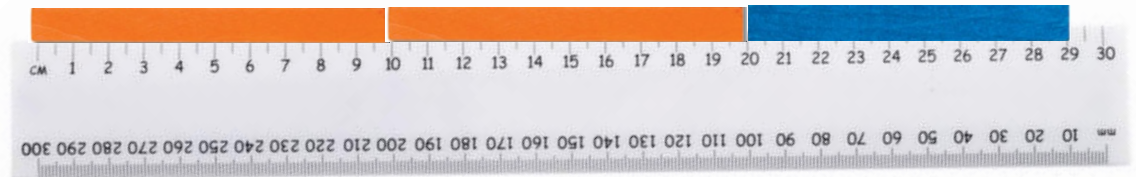
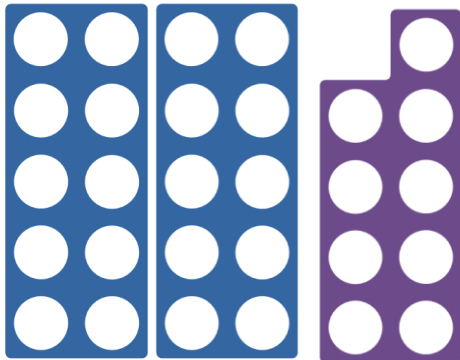


$$10 + 19$$



$$20 + 9$$

29



$$10 + 10 + 9$$

Introducing tens and ones



Count straws up to ten. (Rec / Year 1)

When we reach ten we can put these together to make 1 ten. Show me 15, 24, 32 etc.

Moves on to Diennes. (Year 1 / 2 / 3)



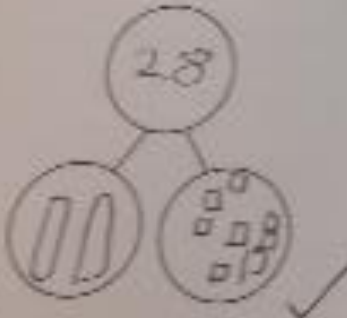
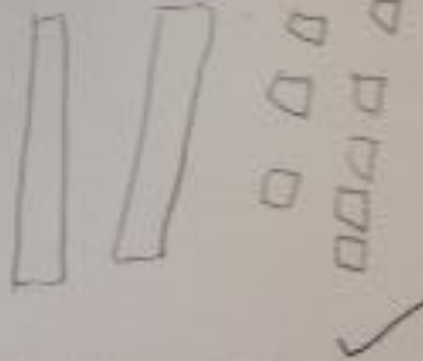
Tens and Ones at home



- *Make up your own systems.*
- *Different coloured bottle tops to represent counters*
- *Tens = sticks, ones = stones*
- *Tens = 1 straw, ones = chopped pieces of straw*
- *Make own version of diennes using cut up strips of paper.*

Moving to Pictorial

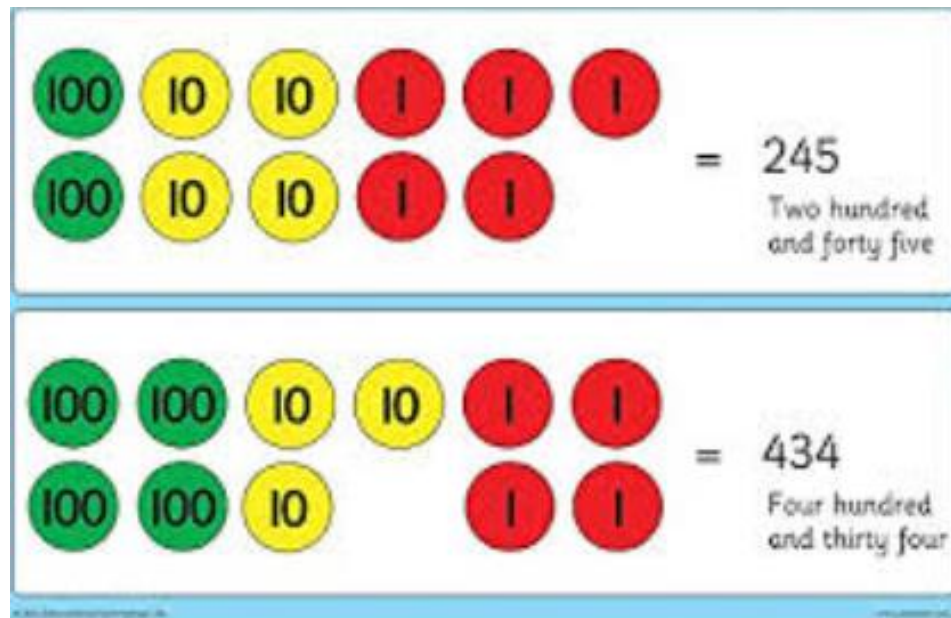


<u>Numerals</u>	<u>Words</u>
28 ✓	twenty eight
<u>Part Whole Model</u>	<u>Draw it</u>
	
Remember the number names! Super. 😊	
YGE Identify and represent numbers pictorially and abstract.	

The next step...



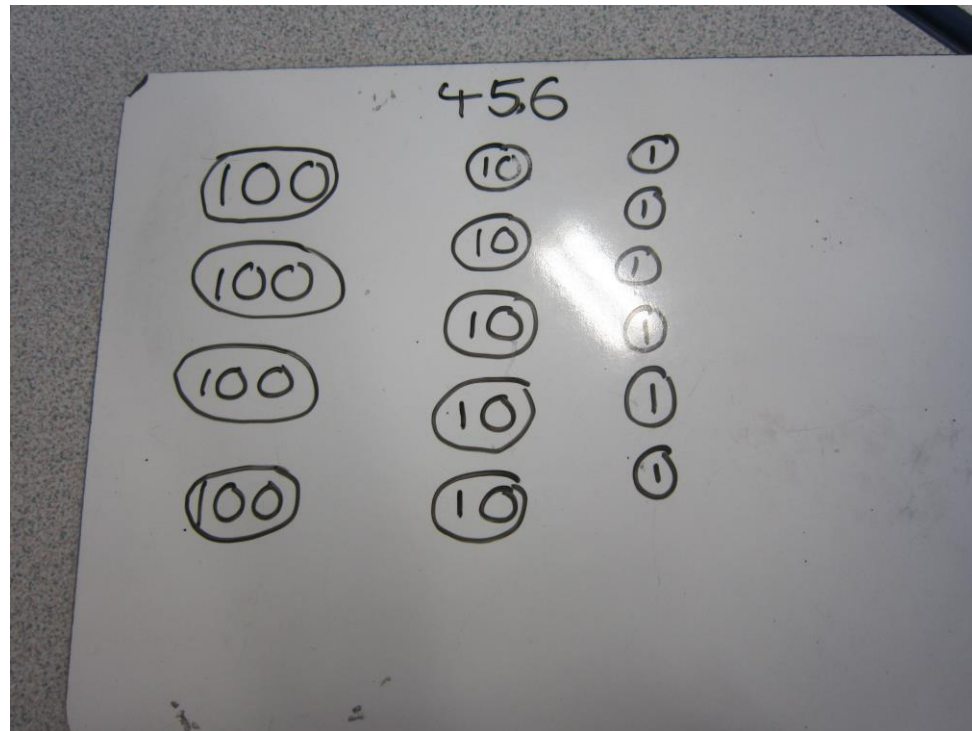
- Moves on to Place Value Counters. (Year 3 / 4 / 5/ 6)



Pictorial Place Value counters



Draw the counters





Addition

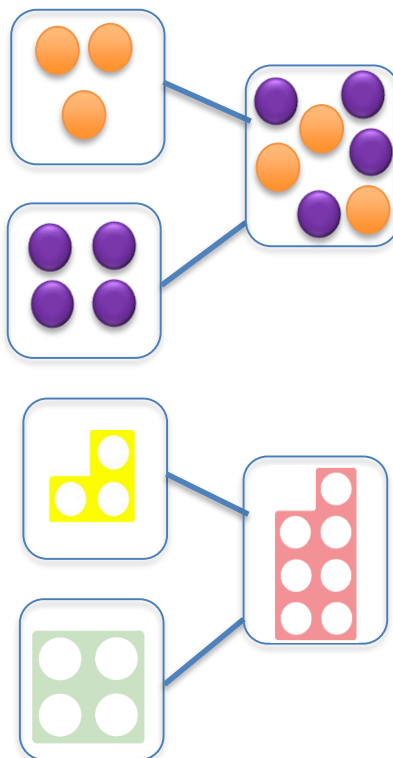
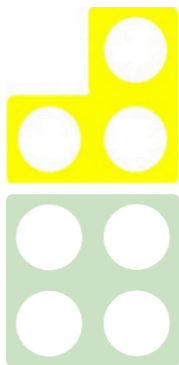
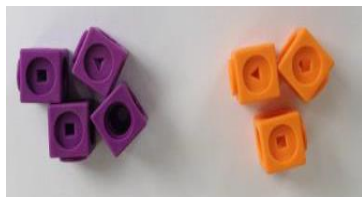
Addition- Part whole model



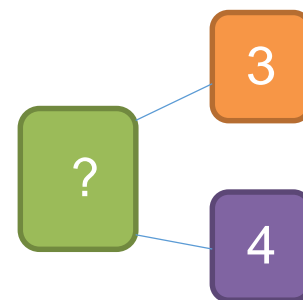
Solve...

$$4 + 3 =$$

Model



Calculations



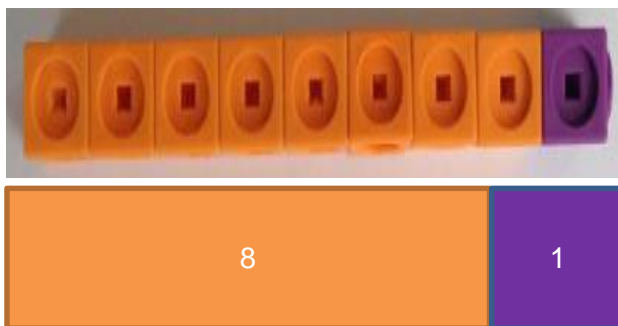
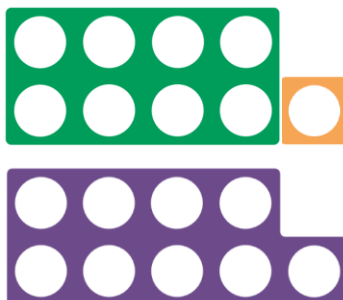
Addition



Solve...

$$8 + 1 =$$

Model



Calculations

$$8 + 1 = 9$$

$$1 + 8 = 9$$

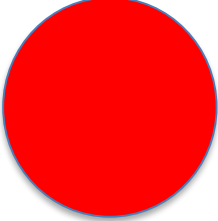
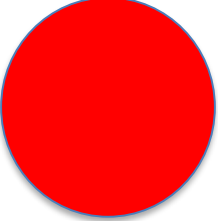
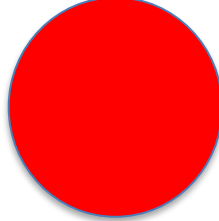
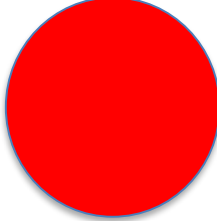

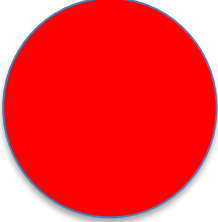
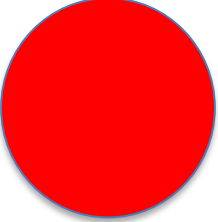
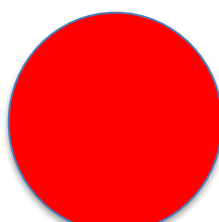
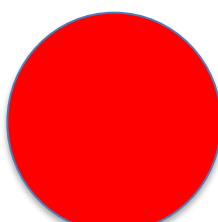

$$8 + \text{[blue square]} = 9$$

$$\text{[blue square]} + 1 = 9$$

Number bonds



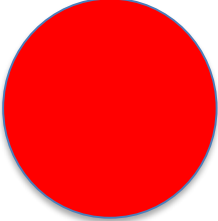
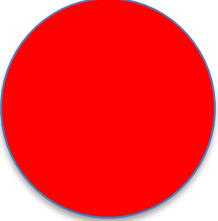
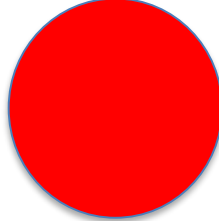
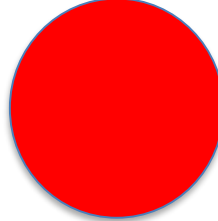

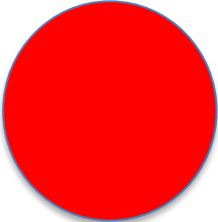




Ten frames- making 10 in different ways

Number bonds



Ten frames- making 10 in different ways

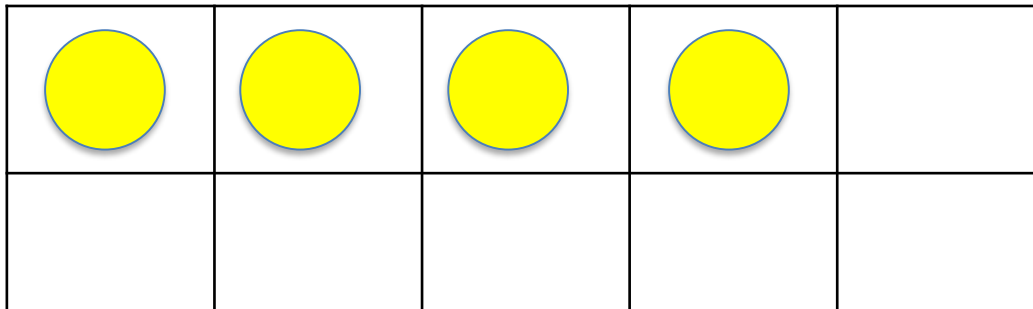
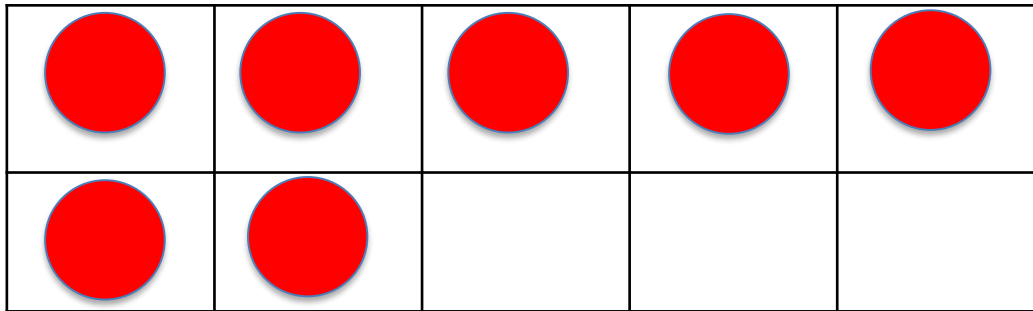
Addition- Regrouping to make 10



Solve...

$$7 + 4$$

Model



Calculations

$$7 + 4 = 11$$

Beginning to use formal written methods

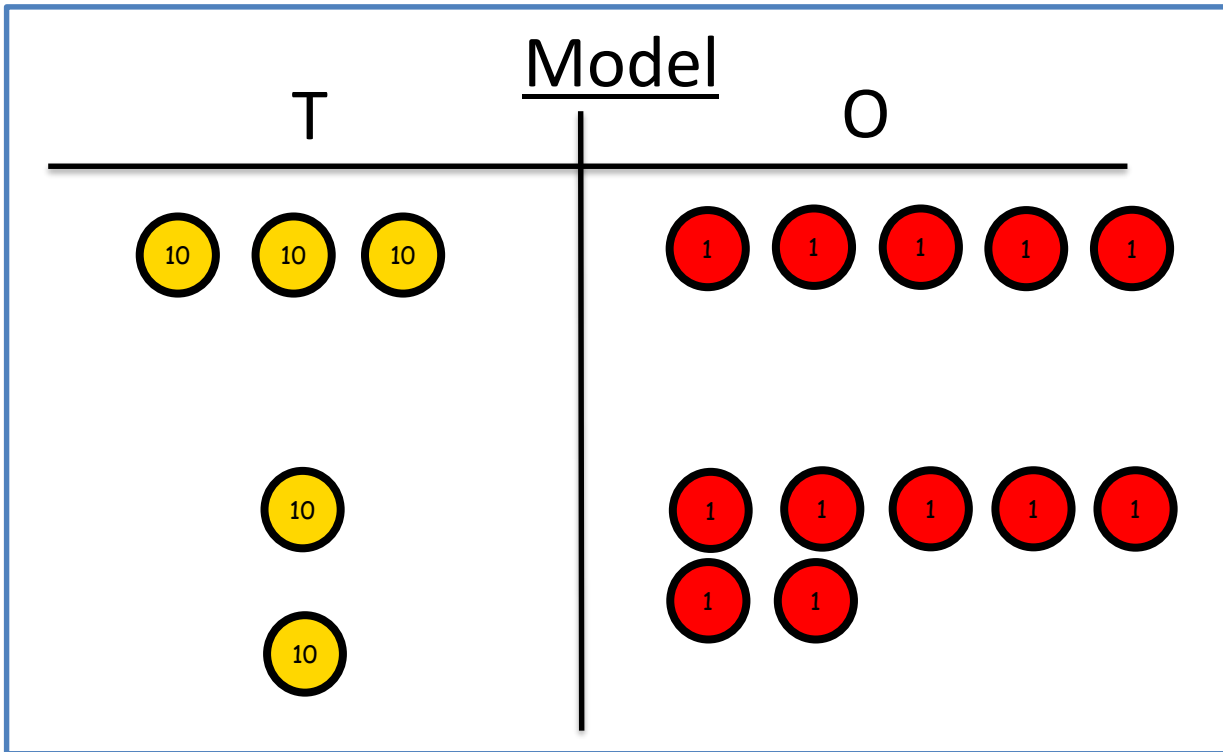


- For the purpose of time all examples use place value counters. However, all of these styles of questions would be done using diennes first until teachers were confident children fully understood the difference between hundreds, tens and ones.



Addition- Column method

Solve... $35 + 17 =$



Calculations

$$\begin{array}{r} 35 \\ +17 \\ \hline 52 \end{array}$$

Key vocabulary: **exchange**

Can we exchange any counters?

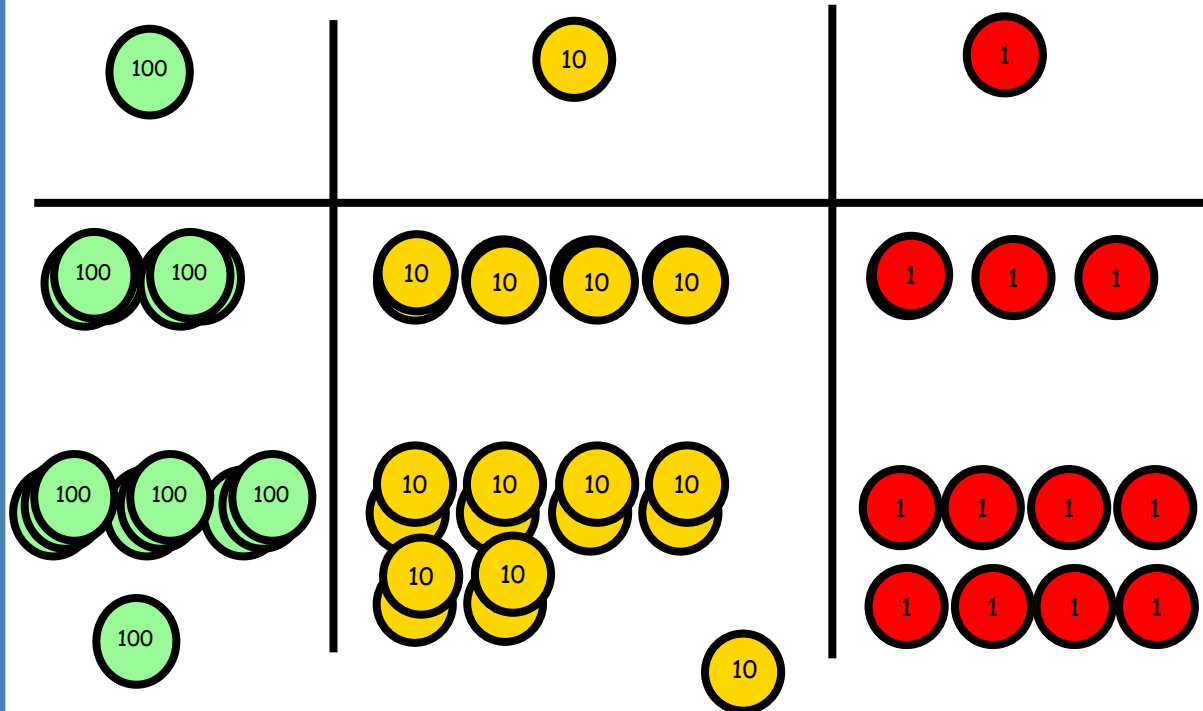


Addition- Column method

Solve...

$$243 + 368 =$$

Model



Calculations

$$\begin{array}{r} 243 \\ +368 \\ \hline 1 \quad 1 \\ \hline 6 \quad 11 \\ \hline \end{array}$$

Key vocabulary: **exchange**

Can we exchange any counters?

Moving to pictorial



After lots of experience with the actual counters children should be able to draw the place value counters to help them solve a calculation, crossing out any counters that are to be exchanged.



Subtraction

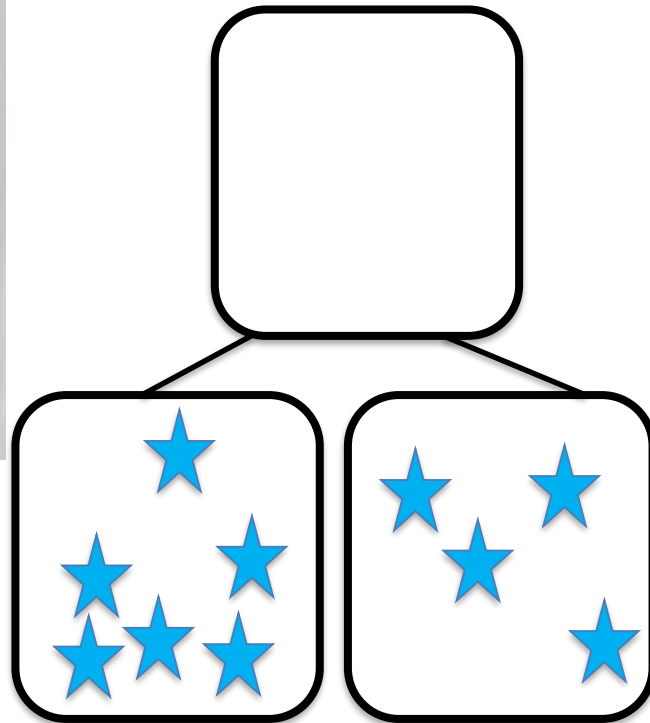
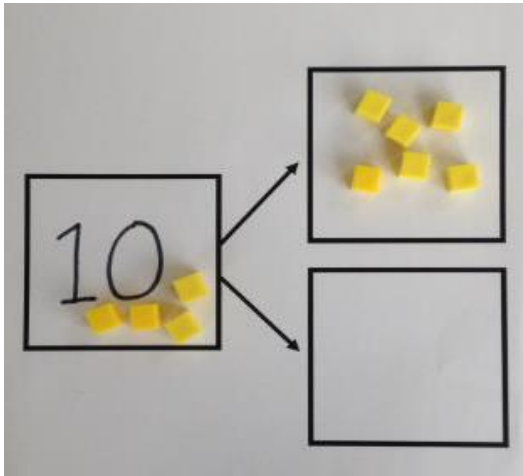
Subtraction- Part whole model



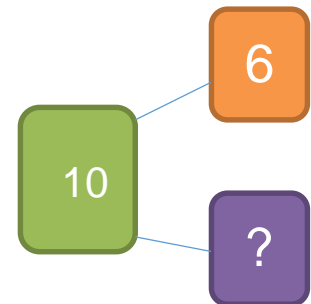
Solve...

$$10 - 6 =$$

Model



Calculations



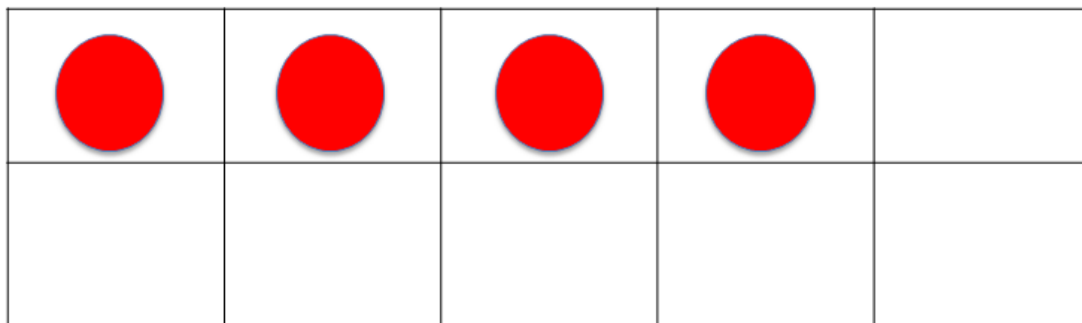
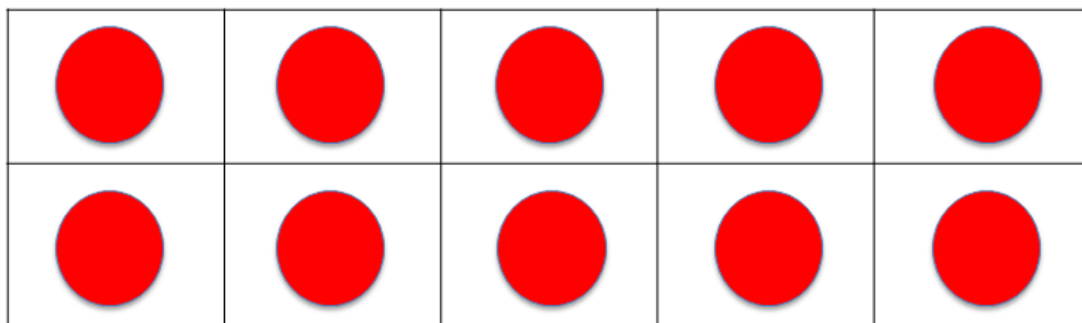
Subtraction - make 10



Solve...

$$14 - 5 =$$

Model



Calculations

$$14 - 5 =$$

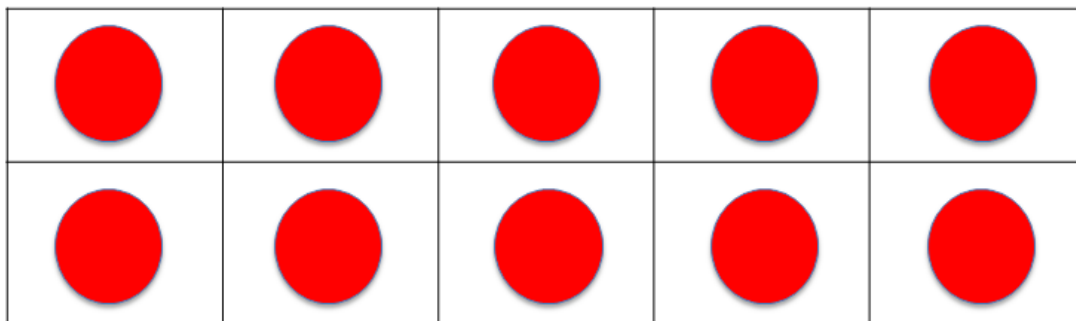
Subtraction - make 10



Solve...

$$14 - 5 =$$

Model



Calculations

$$14 - 5 =$$

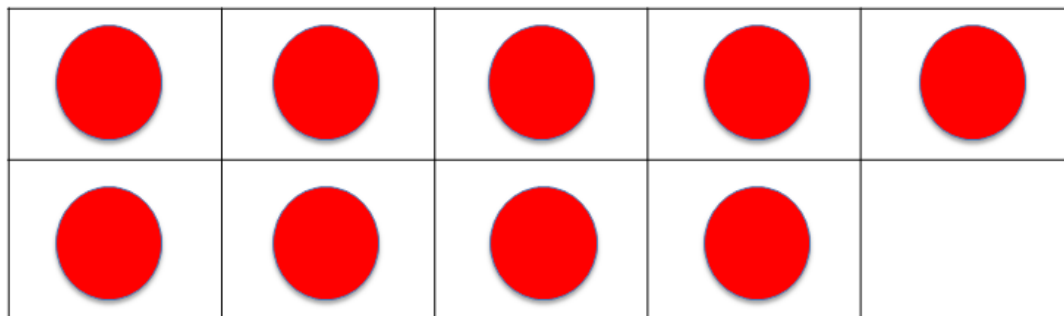
Subtraction - make 10



Solve...

$$14 - 5 =$$

Model



Calculations

$$14 - 5 = 9$$

Subtraction- column method

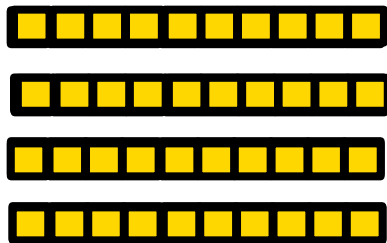


Solve...

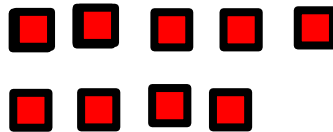
$$49 - 27 =$$

Model

T



O



Calculations

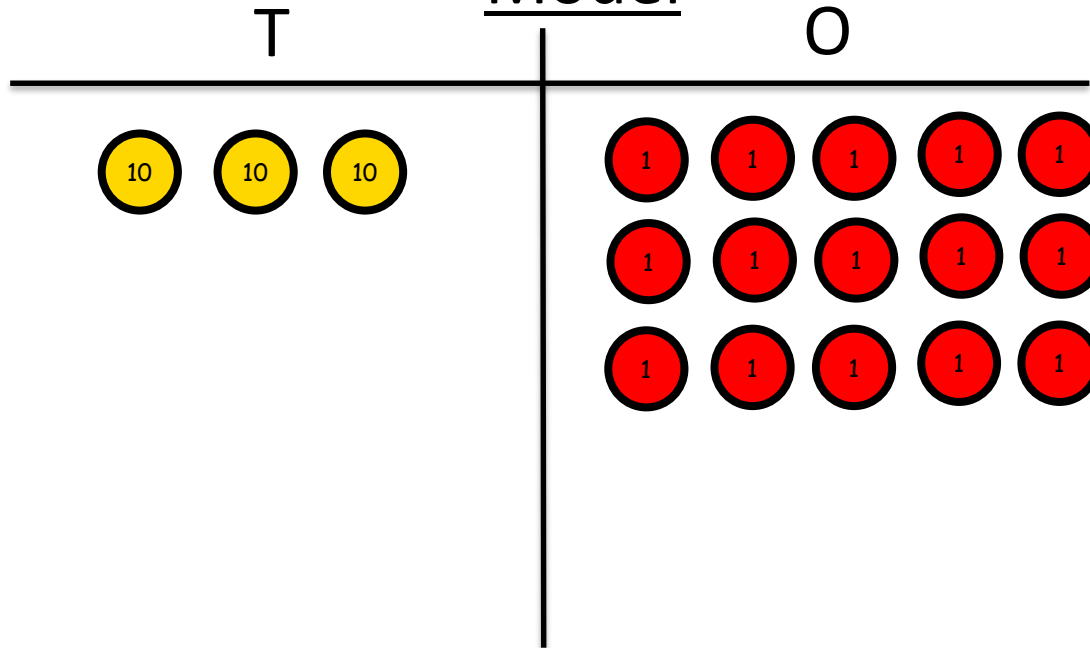
$$\begin{array}{r} 49 \\ -27 \\ \hline 22 \\ \hline \end{array}$$



Subtraction- Column method

Solve... $35 - 17 =$

Model



Calculations

$$\begin{array}{r} 2\cancel{3}^15 \\ -17 \\ \hline 18 \end{array}$$

Key vocabulary: **exchange**

Can we exchange any counters?

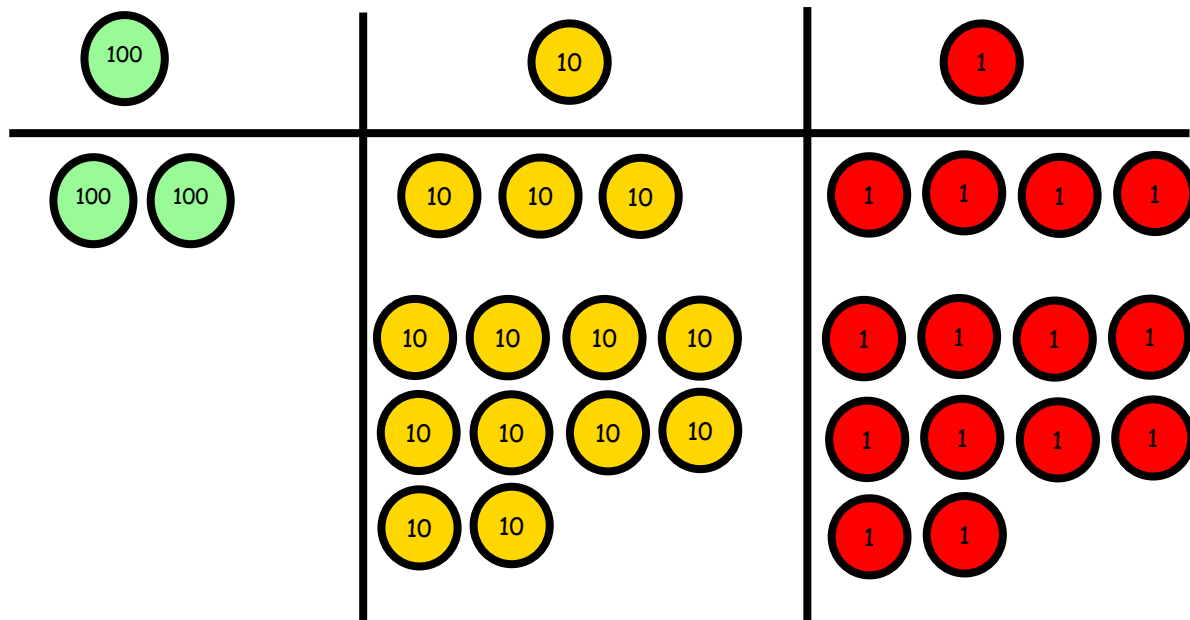
Subtraction- column method



Solve...

$$234 - 88 =$$

Model



Calculations

$$\begin{array}{r} \overset{1}{\cancel{2}} \overset{1}{\cancel{3}} \overset{1}{4} \\ - \quad 88 \\ \hline 146 \end{array}$$

Key vocabulary: **exchange**

Can we exchange any counters?



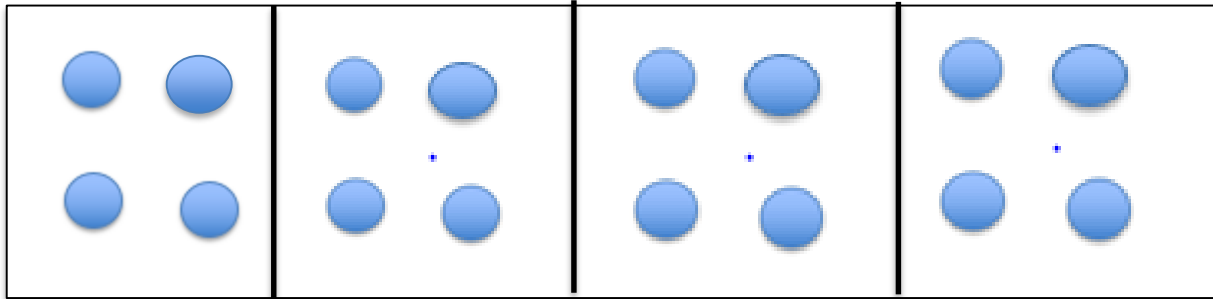
Multiplication



Multiplication using concrete

16

$$4 \times 4 =$$



4

8

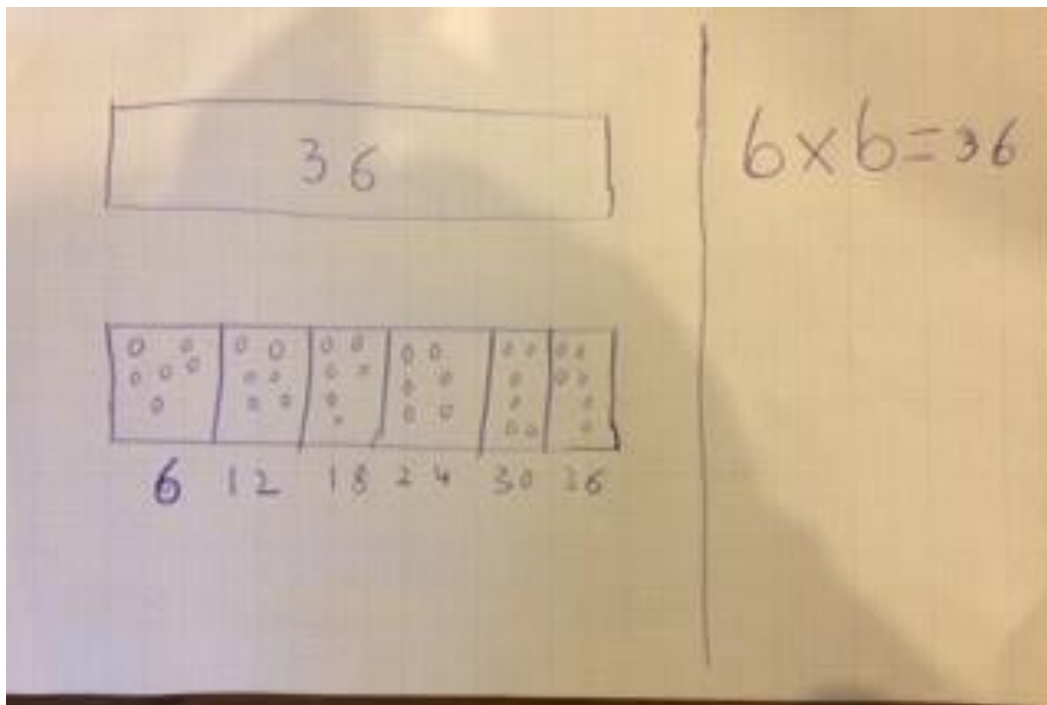
12

16

Pictorial with Place Value Counters



Children very quickly move on to using pictures to represent the place value counters and this can easily be done at home too.



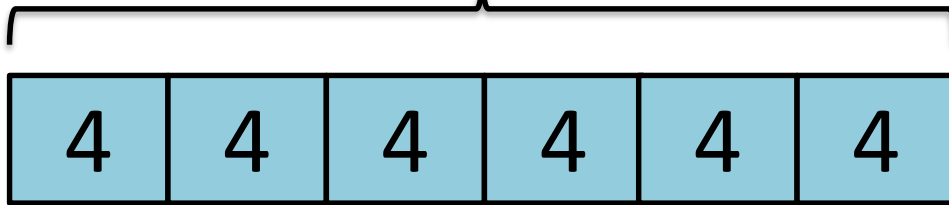
Multiplication



Muffins come in boxes of 4. Peter buys 6 boxes of muffins.
How many muffins does Peter buy all altogether?

Model

?



Calculations

$$6 \times 4 = 24$$



Moving to 2 digit numbers

Place value counters come into place again when we want to multiply 2 digit numbers.

$6 \times 23 =$

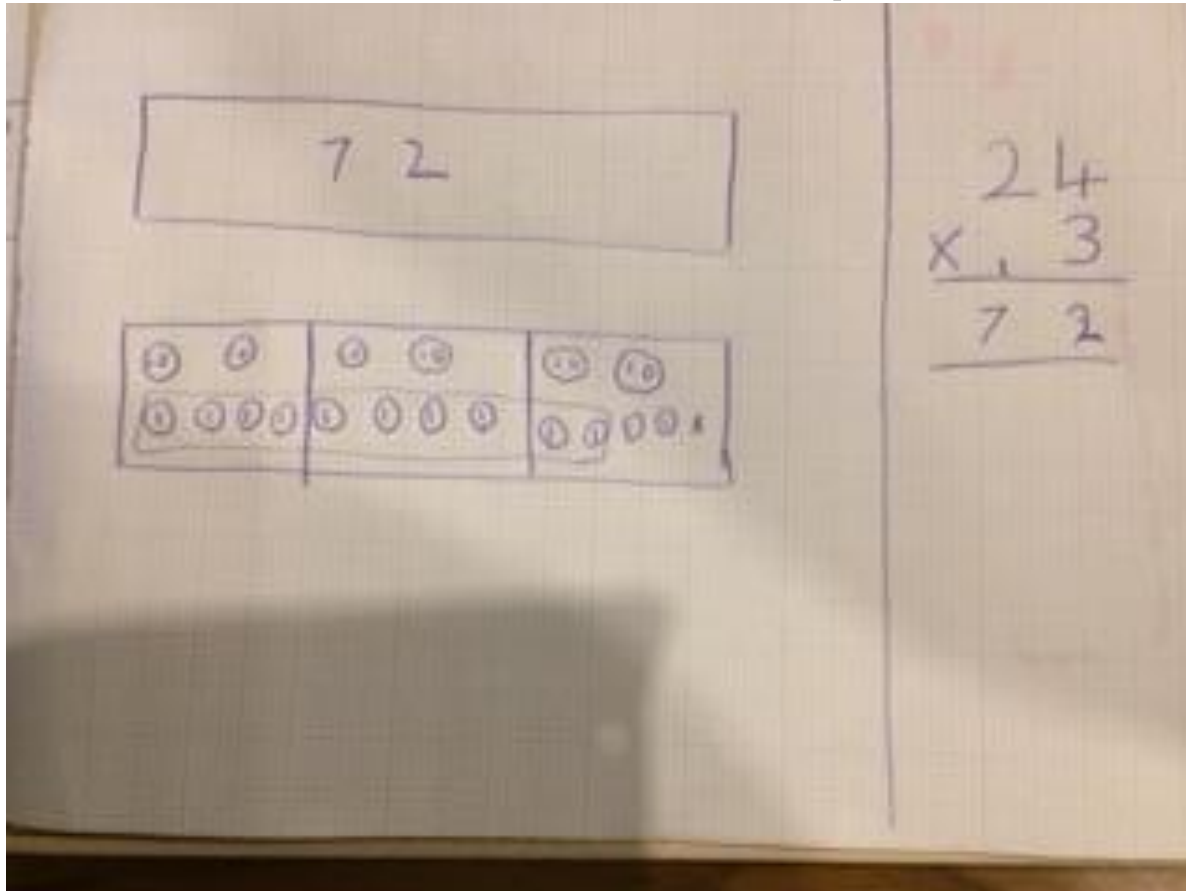
A place value chart with four columns labeled 'Tens' and 'Ones' from left to right. The 'Tens' column contains 6 red counters, and the 'Ones' column contains 18 white counters, representing the number 62.

Calculations

$$\begin{array}{r} 23 \\ \times 6 \\ \hline \end{array}$$



Pictorial with 2 digits

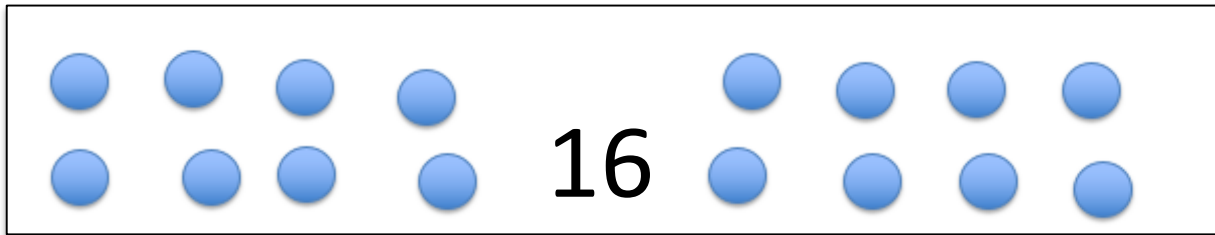




Division



Division using concrete



$$16 \div 4 =$$

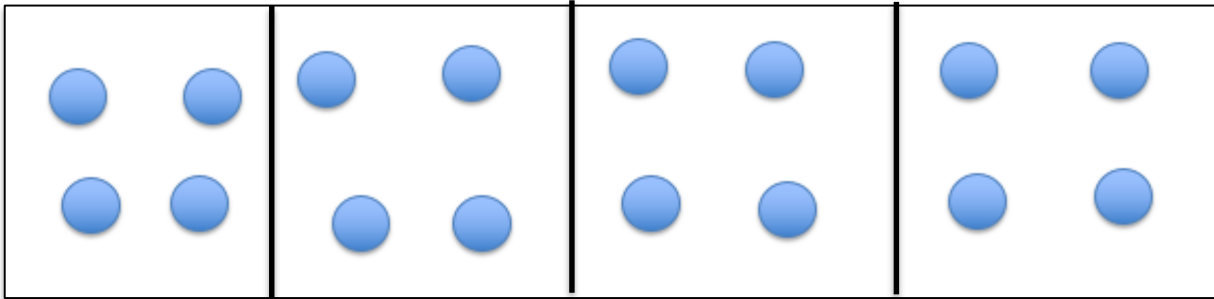
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Division using concrete

16

$$16 \div 4 = 4$$

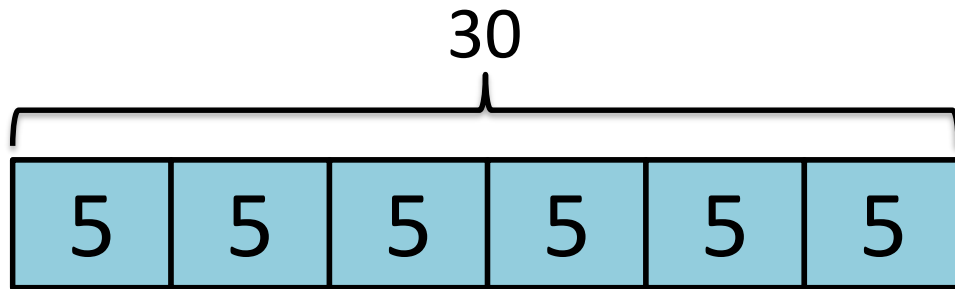


Division (grouping)



Jane has 30 cakes. She wants to pack them into boxes with 5 cakes in each box. How many boxes will she need?

Model



Number of boxes needed = 6

Calculations

$$30 \div 5 = 6$$

In this version, we are counting how many fives go into thirty.

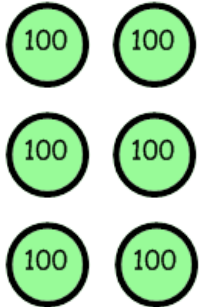

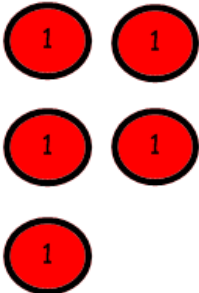
Division



Solve...

$$615 \div 5 =$$

Model

H	T	O
		

Calculations

$$5 \overline{) 615}$$

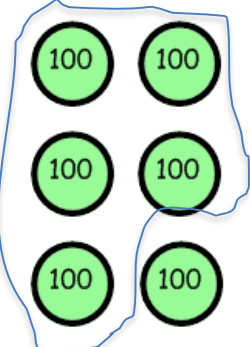

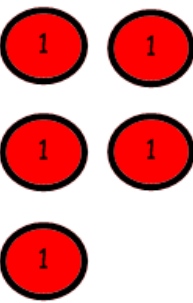
Division



Solve...

$$615 \div 5 =$$

Model

H	T	O
		

Calculations

$$\begin{array}{r} 1 \\ 5 \overline{) 615} \end{array}$$

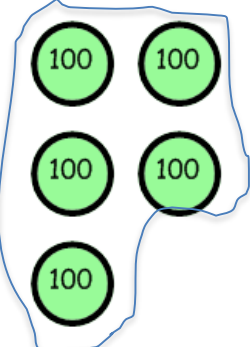
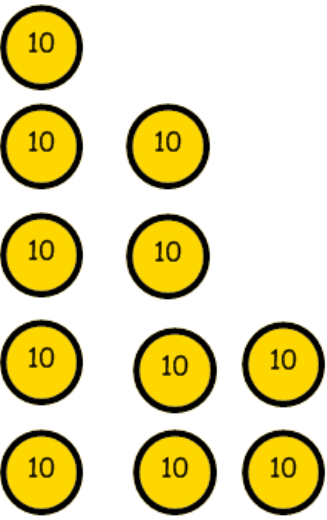
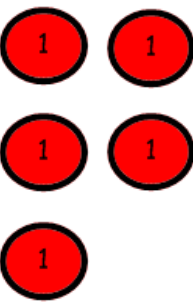
Division



Solve...

$$615 \div 5 =$$

Model

H	T	O
		

Calculations

$$\begin{array}{r} 1 \\ 5 \overline{) 615} \end{array}$$

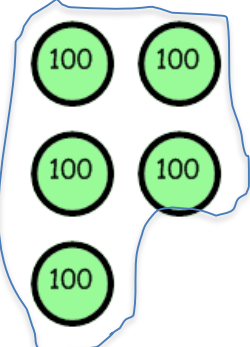
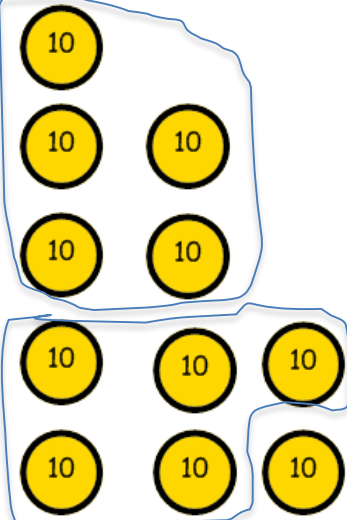
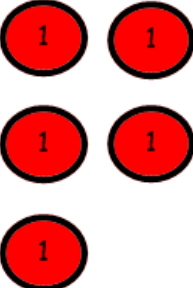
Division



Solve...

$$615 \div 5 =$$

Model

H	T	O
		

Calculations

$$\begin{array}{r} 12 \\ 5 \overline{) 615} \end{array}$$

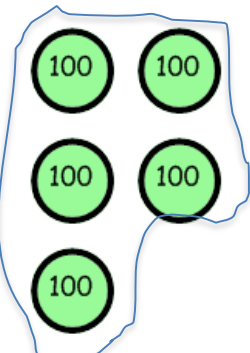
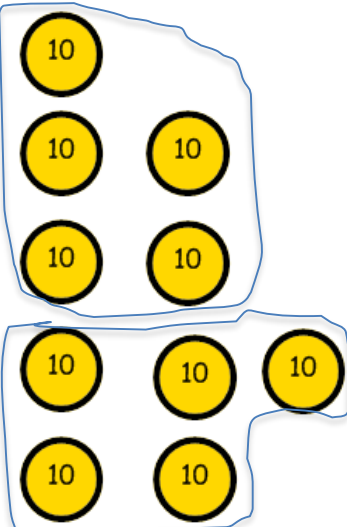
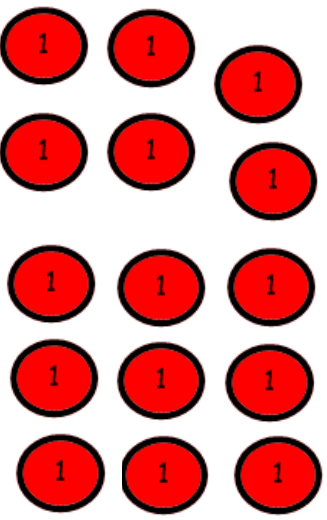
Division



Solve...

$$615 \div 5 =$$

Model

H	T	O
		

Calculations

$$\begin{array}{r} 12 \\ 5 \overline{) 615} \\ \underline{5} \\ 11 \\ \underline{10} \\ 15 \\ \underline{15} \\ 0 \end{array}$$

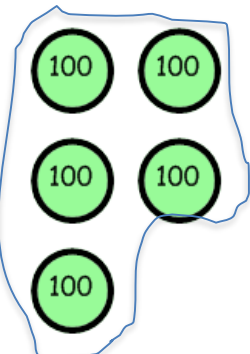
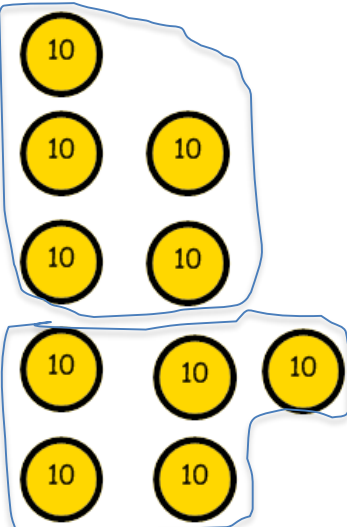
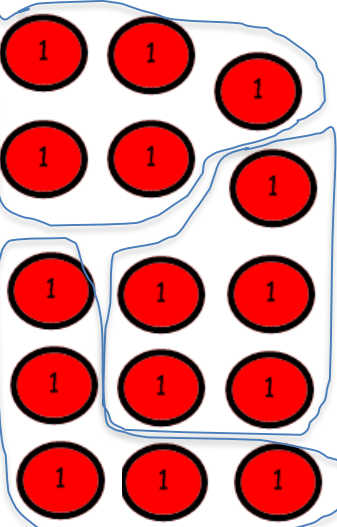
Division



Solve...

$$615 \div 5 =$$

Model

H	T	O
		

Calculations

$$\begin{array}{r} 123 \\ 5 \overline{) 615} \end{array}$$



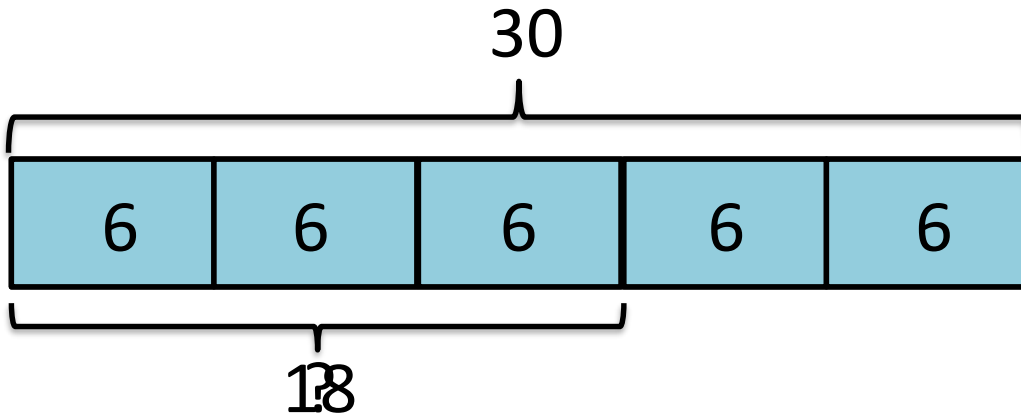
Fractions

Fraction of an Amount



Peter starts with 30 sweets. He eats $\frac{3}{5}$ of them. How many sweets does he eat?

Model



Calculations

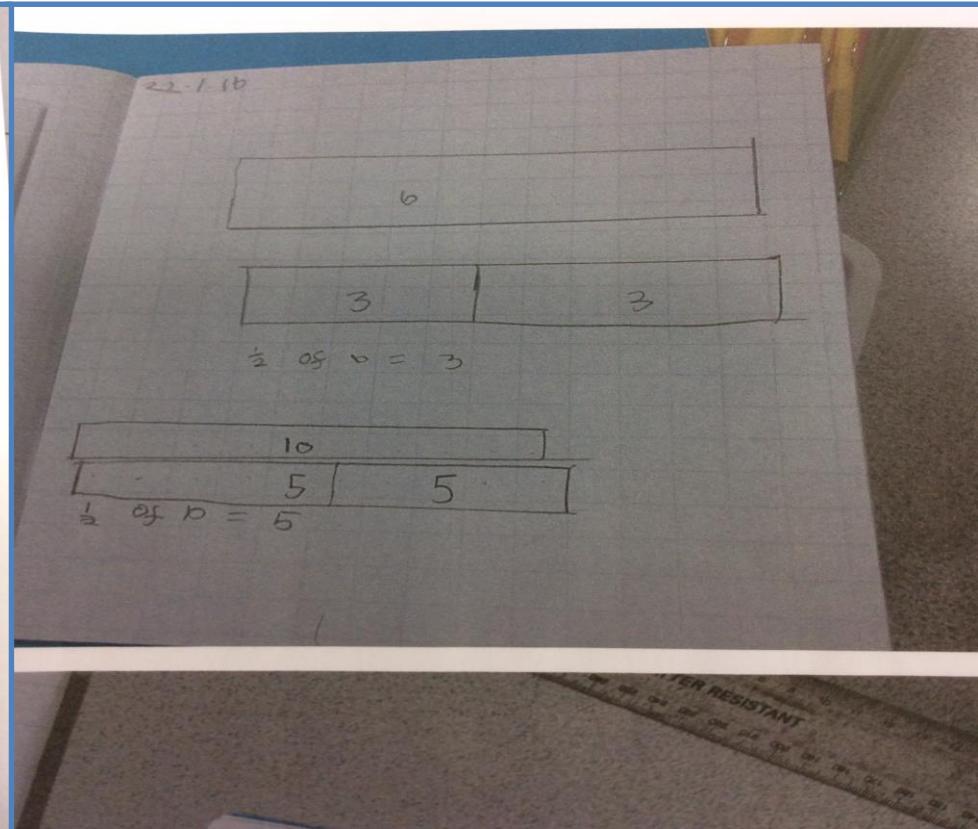
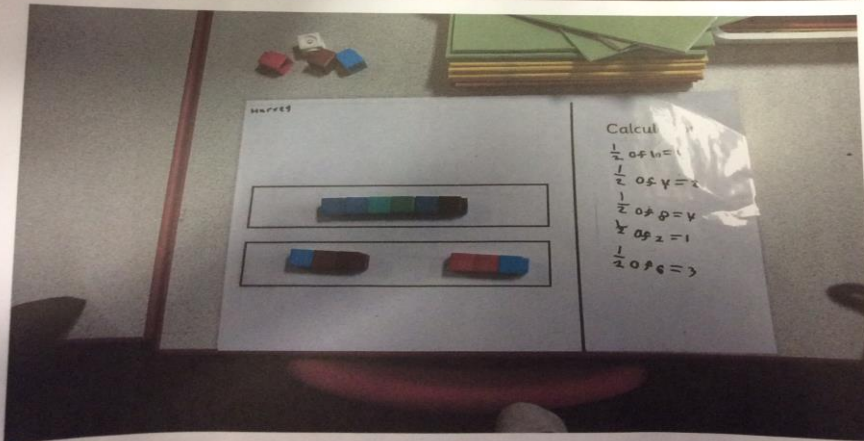
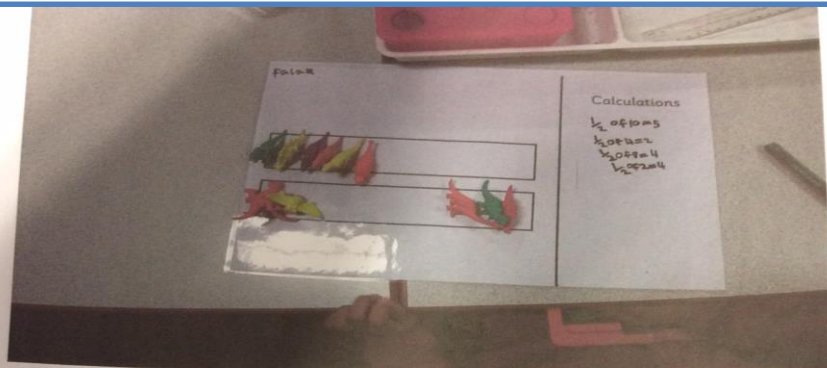
$$30 \div 5 = 6$$

$$6 \times 3 = 18$$

Children's example



Year 1



Children's example



Year 3

as I use bar modelling for \times and \div ✓

16 \div 2 = 8 ✓

36 \div 3 = 12 ✓

12 12 12

336 \div 12 = 28 ✓

12 \times 3 = 36 ✓

3 \times 12 = 36 ✓

4 \times 8 = 32 ✓

4 4 4 4 4 4 4 4

48 12 16 20 24 28 32

8 \times 4 = 32 ✓

32 \div 4 = 8 ✓

32 \div 8 = 4 ✓

2 \times 9 = 18 ✓

18 \div 2 = 9 ✓

18 \div 9 = 2 ✓

50 \div 5 = 10 ✓

50 \div 10 = 5 ✓

5 \times 10 = 50 ✓

55 55 55 55 55

5 \times 5 = 25 ✓

Can I find non unit fractions of amounts? ✓ Fab! Try the deeper activity

Sam eats $\frac{3}{5}$ of a bag of sweets. There are 35 sweets. How many did he eat?

35

35 \div 5 = 7

3 \times 7 = 21

$\frac{3}{5}$ = 21 ✓

Harry has 40 stickers in his collection. $\frac{3}{4}$ of them are football ones. How many are football stickers?

40

40 \div 4 = 10

3 \times 10 = 30

$\frac{3}{4}$ = 30 ✓

A footballer scores 20 goals in a season. $\frac{3}{5}$ of the goals came from heading the ball. How many goals did he score from heading the ball?

20

20 \div 5 = 4

3 \times 4 = 12

$\frac{3}{5}$ = 12 ✓

Mrs Chapman walks for 15 miles. After $\frac{2}{3}$ of the journey she stops for a rest. How many miles had she walked before she stopped?

15

15 \div 3 = 5

2 \times 5 = 10

$\frac{2}{3}$ = 10 ✓

Year 3 bake 28 cupcakes for a charity sale. They sell $\frac{4}{7}$ of them during morning playtime. How many did they sell?

28

28 \div 7 = 4

4 \times 4 = 16

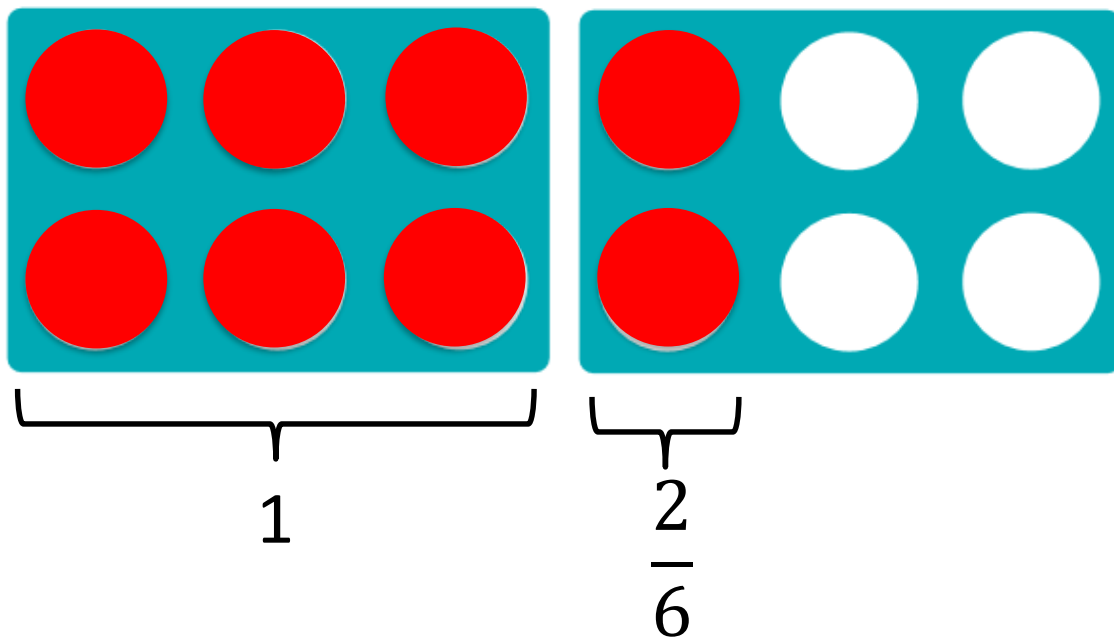
$\frac{4}{7}$ = 16 ✓

Converting fractions



Convert $\frac{8}{6}$ into a mixed number fraction

Model



Calculations

$$\frac{8}{6} = 1 \frac{2}{6}$$

Ratio



Peter and Jane share £40 in the ratio of 3:5

How much money does each person get?

Model

Peter



Jane



£40

Calculations

$$40 \div 8 = 5$$

$$\text{Peter: } 5 \times 3 = 15$$

$$\text{Jane: } 5 \times 5 = 25$$

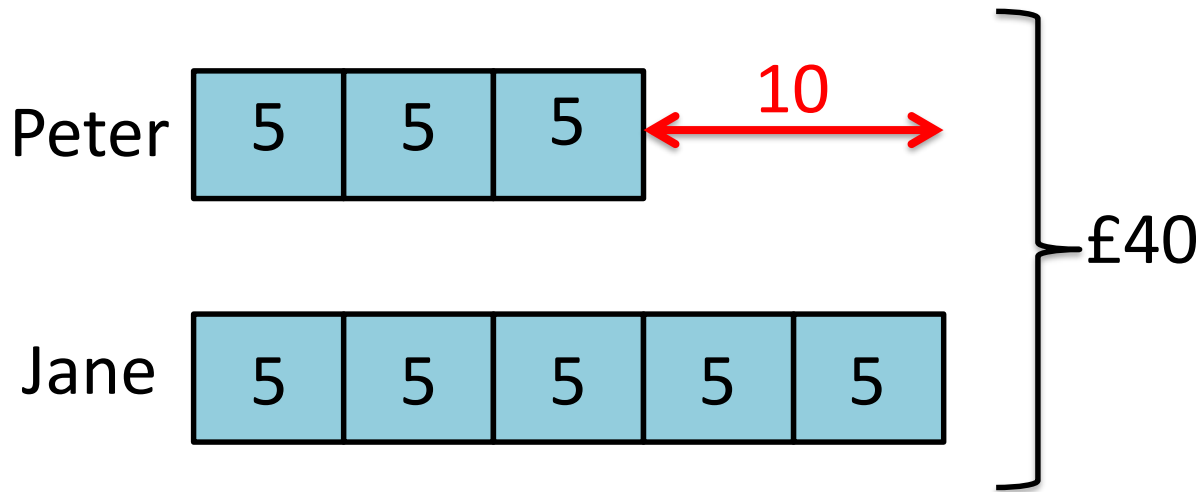
Ratio



Peter and Jane share £40 in the ratio of 3:5

How much more money does Jane have than Peter?

Model



Calculations

$$40 \div 8 = 5$$

$$\text{Peter: } 5 \times 3 = 15$$

$$\text{Jane: } 5 \times 5 = 25$$

$$25 - 15 = 10$$

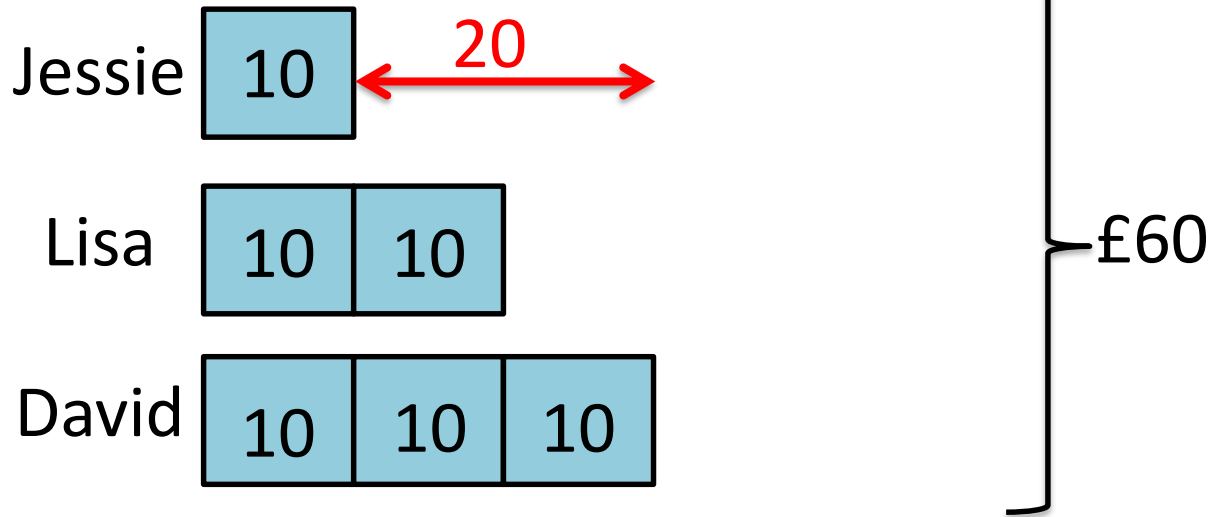
Ratio



Jessie, Lisa and David share £60 in the ratio of 1:2:3

How much more money does David get than Jessie?

Model



Calculations

$$60 \div 6 = 10$$

$$\text{Jessie: } 10 \times 1 = 10$$

$$\text{Lisa: } 10 \times 2 = 20$$

$$\text{David: } 10 \times 3 = 30$$

$$30 - 10 = 20$$



*Thank you for listening.
We hope we have given you a
useful insight into using CPA
approaches within Mathematics.
If you have any questions please
don't hesitate to ask.*