Good evening!

Thank you for coming.

This evening we will provide you with information on how we teach Mathematics at Moorlands and suggest ways that you can help your child at home using similar methods.

Maths at Moorlands.

A focus on Concrete, Pictorial and Abstract approaches. (CPA)

Karen Chapman, Sophie Crowther and Emma Bancroft

Maths Leaders Moorlands Primary School

With thanks to Kelsey Brown, White Rose Maths Hub

Agenda

- The theory behind the importance of CPA
- Using manipulatives to introduce the basics of a new concept and ways to replicate this at home.
- Transitioning between concrete, pictorial and abstract.

Concrete, Pictorial and Abstract Methods

Importance of CPA

In his research on the cognitive development of children (1966), Jerome Bruner proposed three ways of working to aid development:

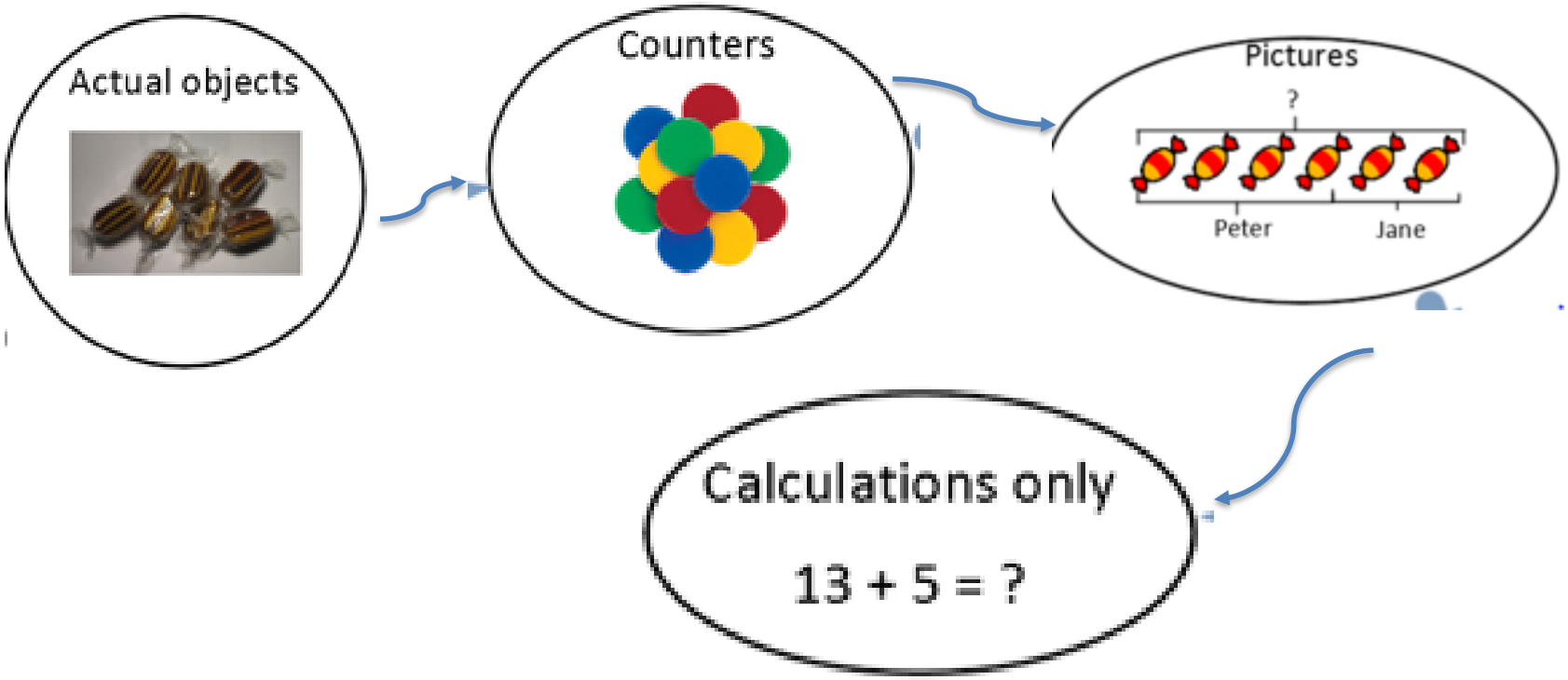
- Enactive representation (using 'concrete' objects)
- Iconic representation (drawing images / pictures)
- Symbolic representation (abstract numbers)

"If we do not use concrete manipulations, then we can not understand mathematics. If we only use concrete manipulations, then we are not doing mathematics."

Gu (2015)

Concrete, Pictorial, Abstract

Children should work at the stage they need until ready to move on. Within a class children can be working on the same calculation but accessing it in different ways.



Using CPA methods

Today we aim to give you a quick insight into methods used at school and how you could adapt those to work at home, with a focus on:

- *Place value*
- *Addition and Subtraction*
- *Multiplication and Division*
- *Fractions and Problem Solving*

Place Value

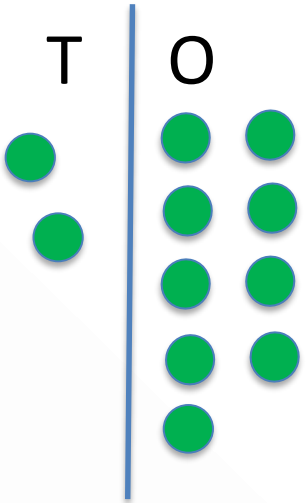
(understanding the value of each digit and it's place in the number system)

Place value

How many ways can you show 29?

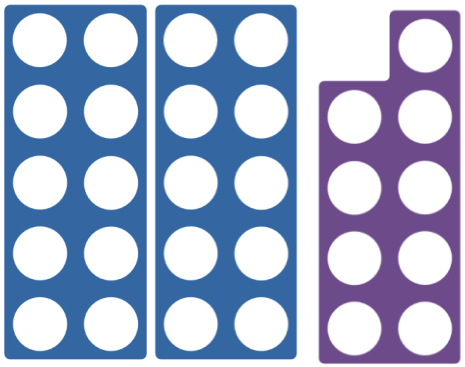
Twenty nine

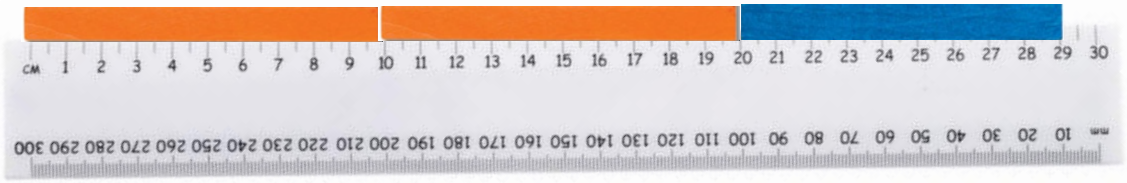
10 + 19



20 + 9

29





10 + 10 + 9

Introducing tens and ones

Count straws up to ten. (Rec / Year 1)

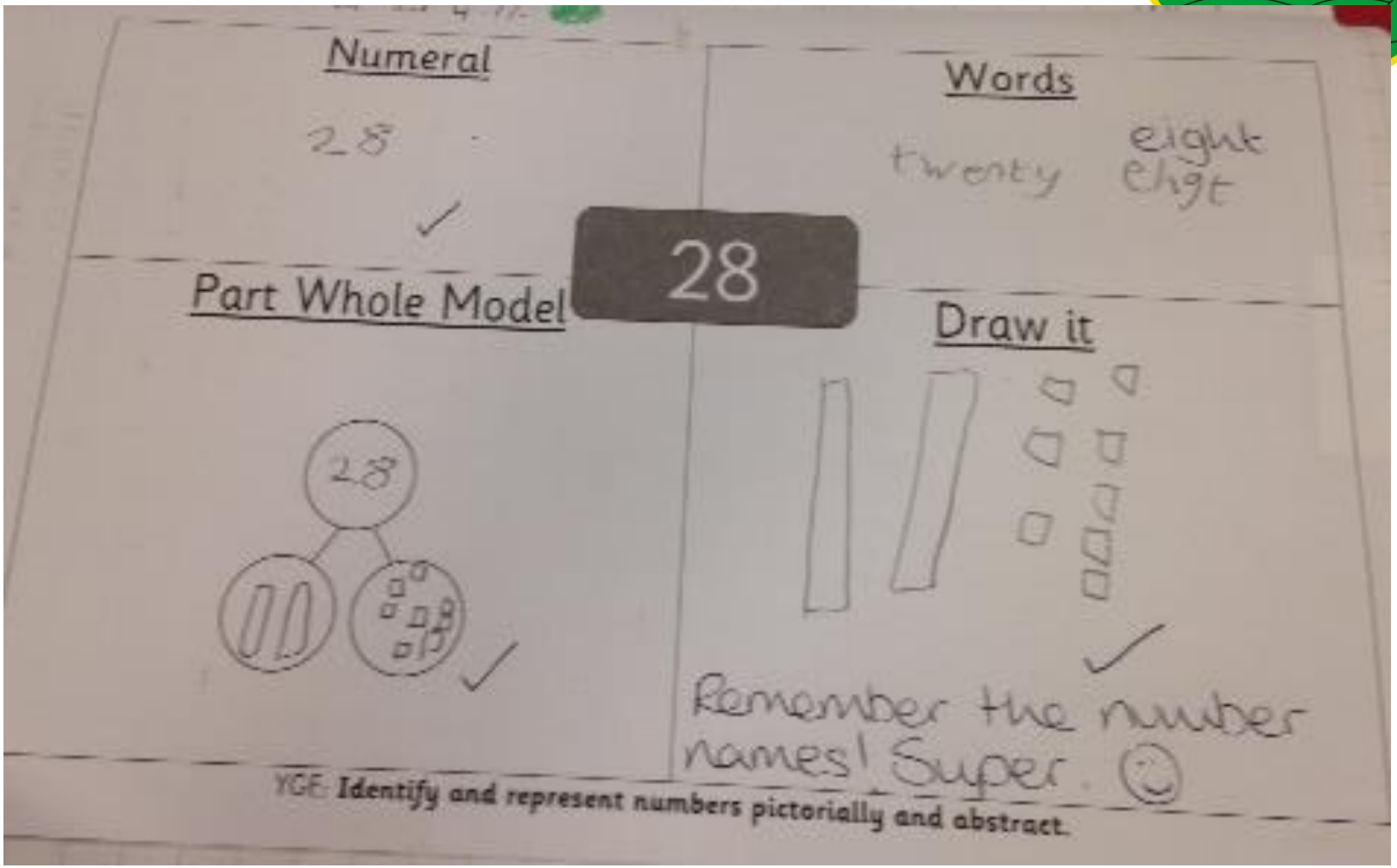
When we reach ten we can put these together to make 1 ten. Show me 15, 24, 32 etc.

Moves on to Diennes. (Year 1 / 2 / 3)

Tens and Ones at home

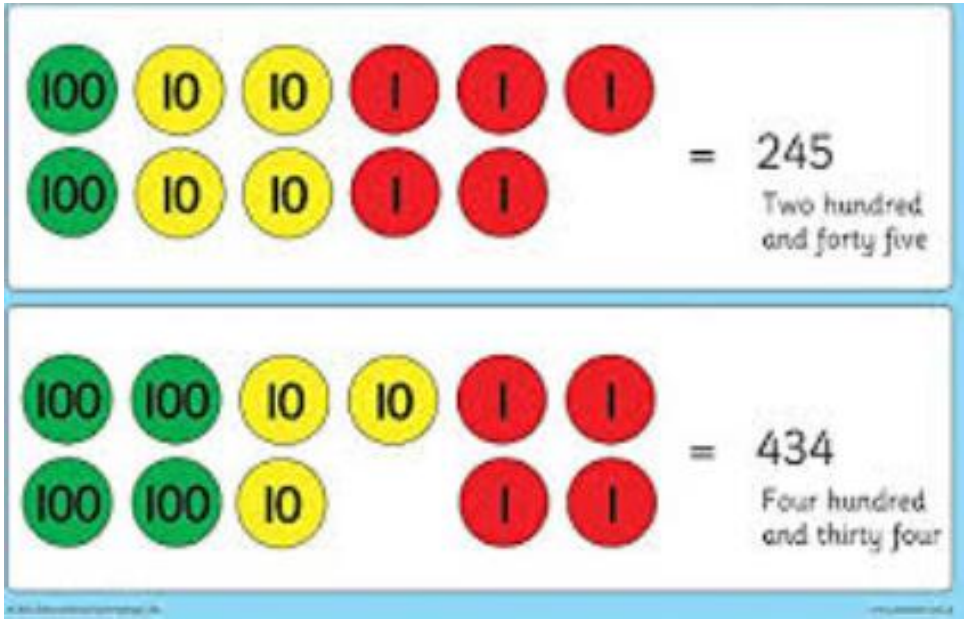
- Make up your own systems.
- Different coloured bottle tops to represent counters
- Tens = sticks, ones = stones
- Tens = 1 straw, ones = chopped pieces of straw
- Make own version of diennes using cut up strips of paper.

Moving to Pictorial



The next step…

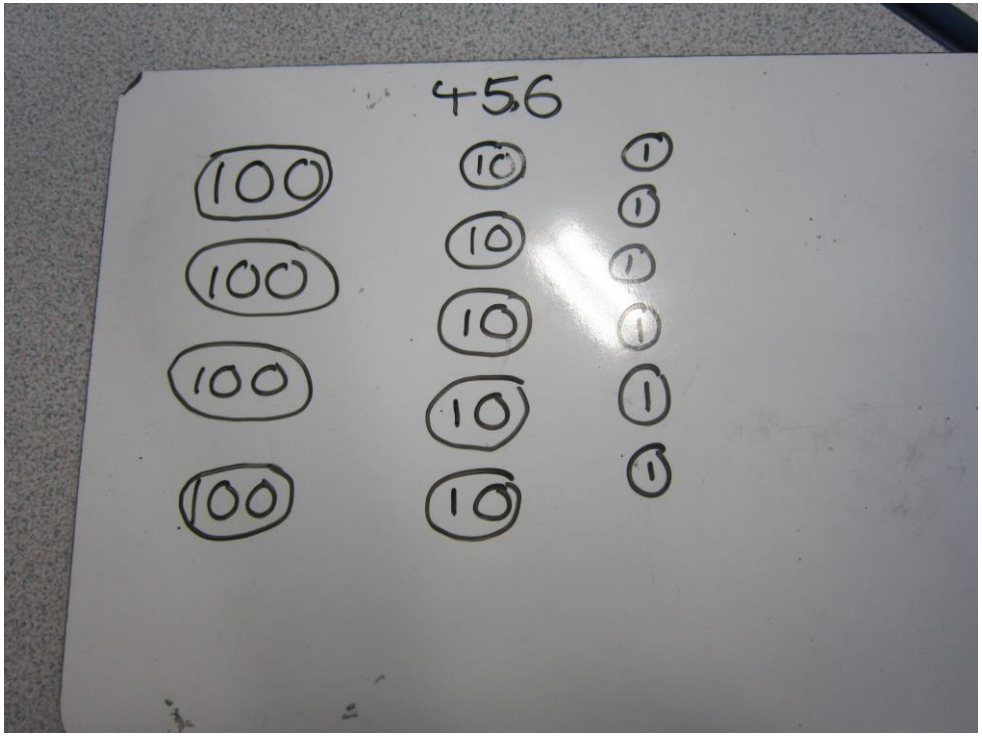
- Moves on to Place Value Counters. (Year 3 / 4 / 5/ 6)



Pictorial Place Value counters

Moorlands Primary School

Draw the counters

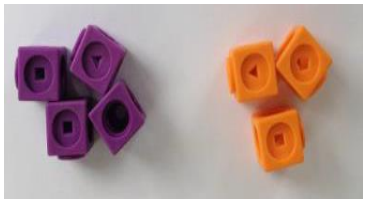


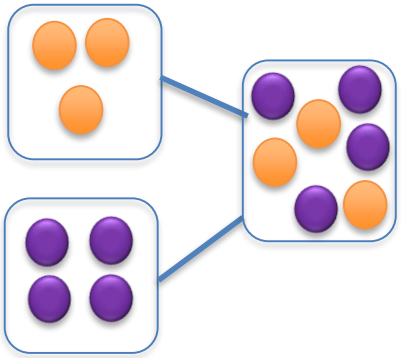
Addition

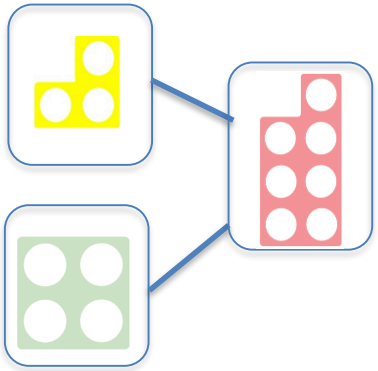
Addition- Part whole model

Solve… 4 + 3 =

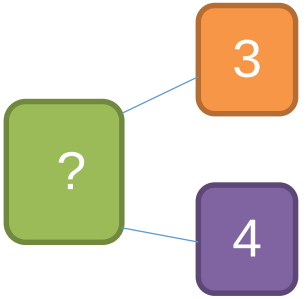
Model







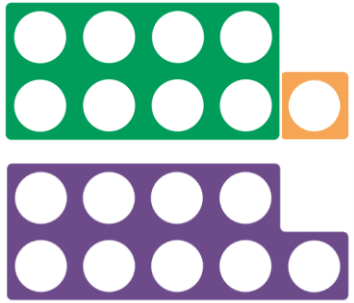
Calculations

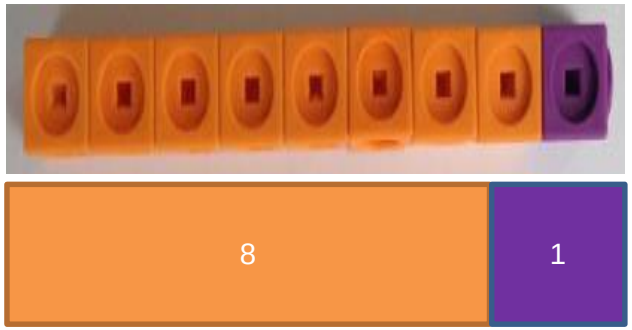


Addition

Solve… 8 + 1 =

Model





Calculations

8 + 1 = 9

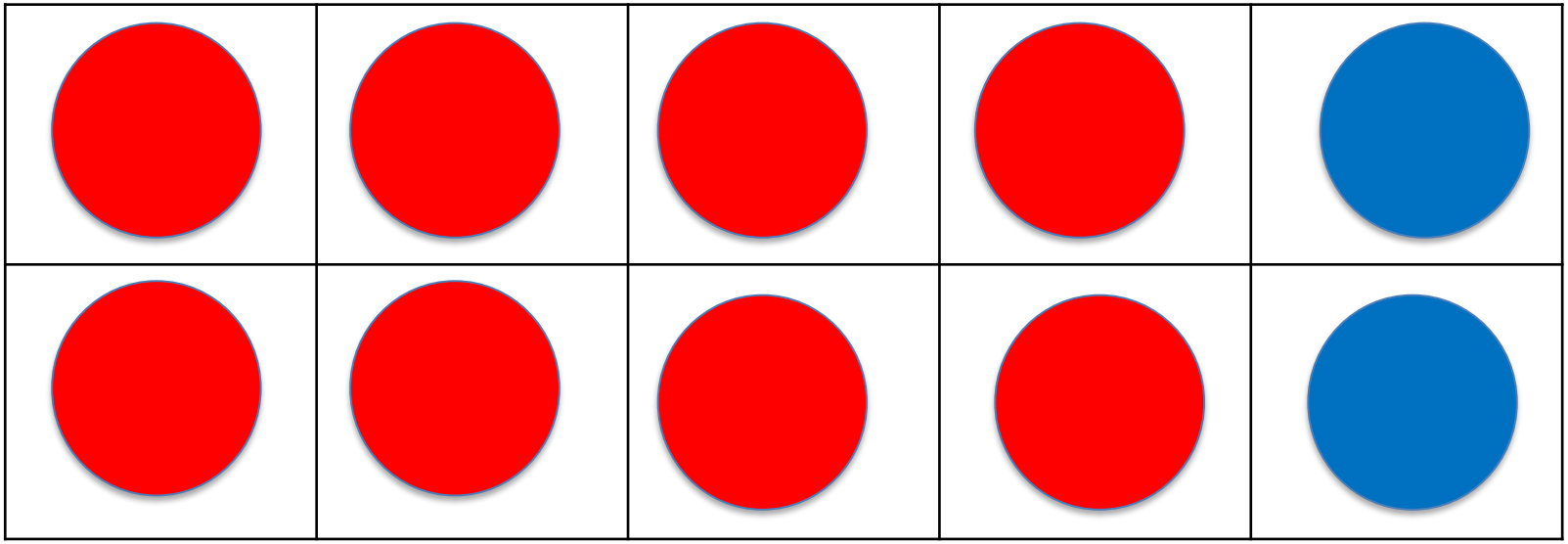
1 + 8 = 9

8 + ▢ = 9

▢ + 1 = 9

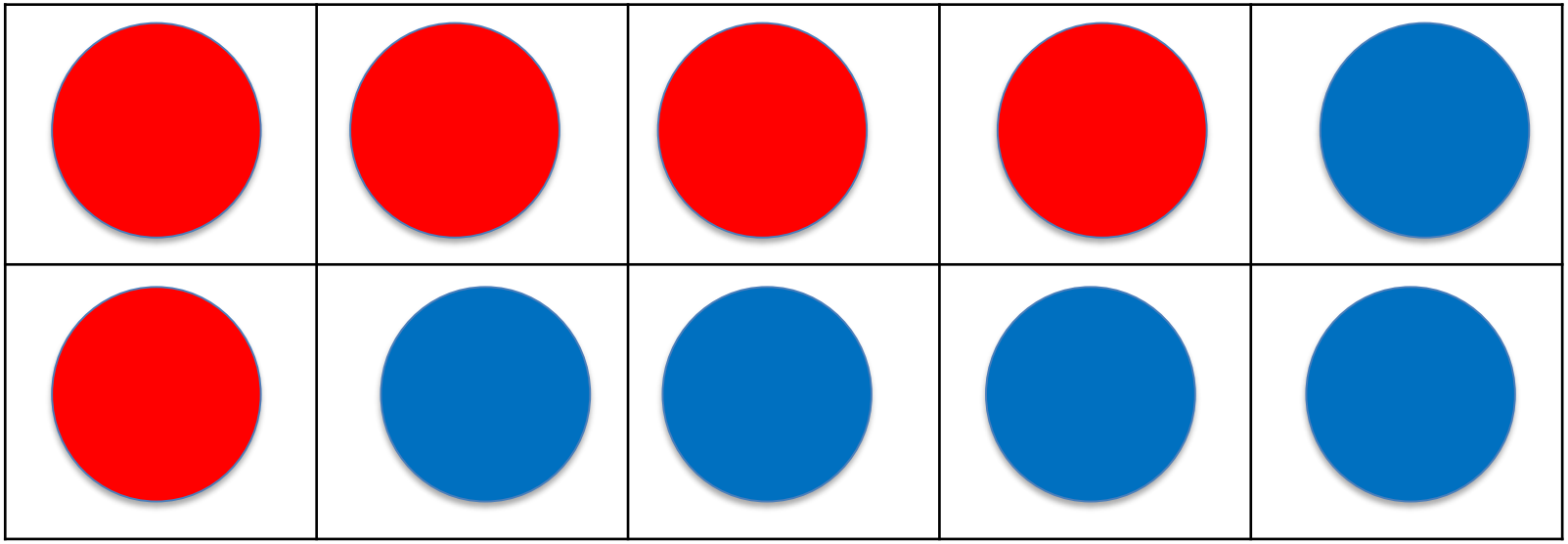
Number bonds

Ten frames- making 10 in different ways



Number bonds

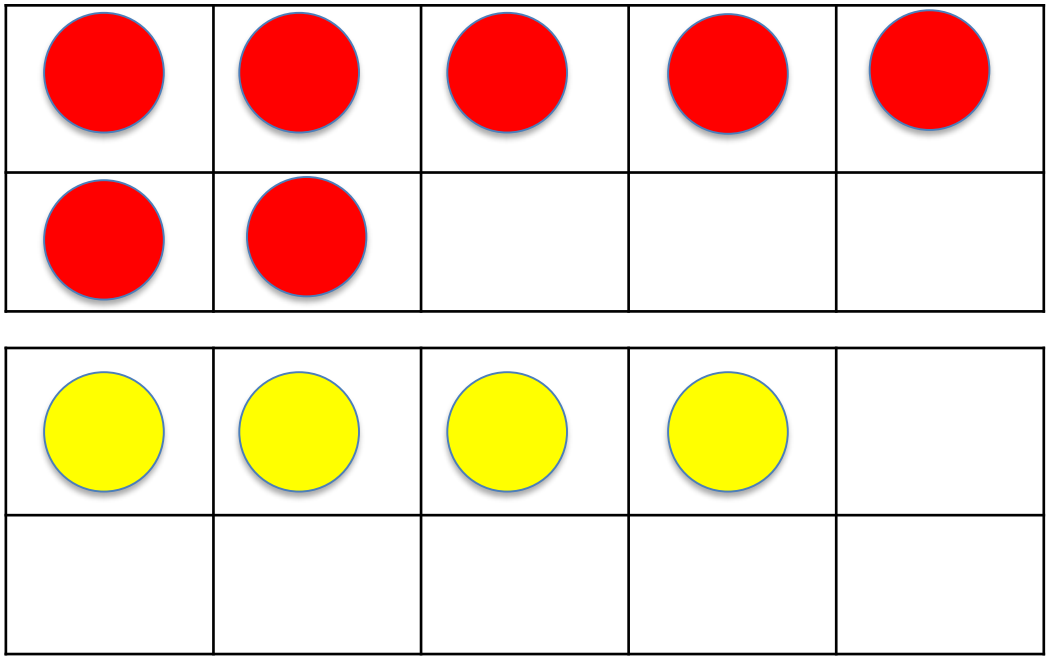
Ten frames- making 10 in different ways



Addition- Regrouping to make 10

Solve... 7 + 4

Model



Calculations

7 + 4 = 11

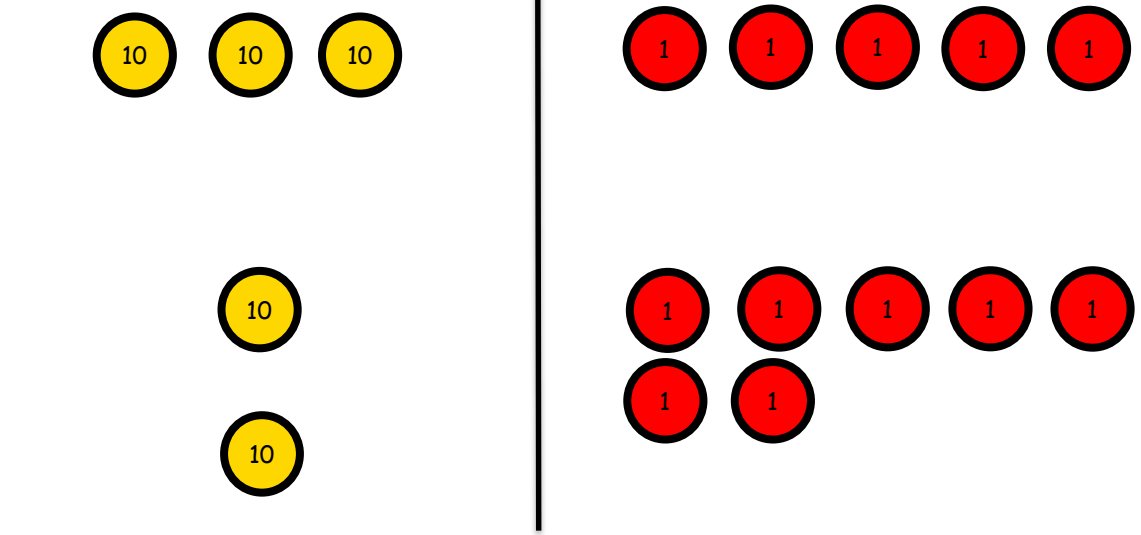
Beginning to use formal written methods

- For the purpose of time all examples use place value counters. However, all of these styles of questions would be done using diennes first until teachers were confident children fully understood the difference between hundreds, tens and ones.

Addition- Column method

Solve… 35 + 17 =

Model



Calculations

$$\begin{array}{r} 35 \\ +17 \\ \hline 52 \\ \hline \end{array}$$

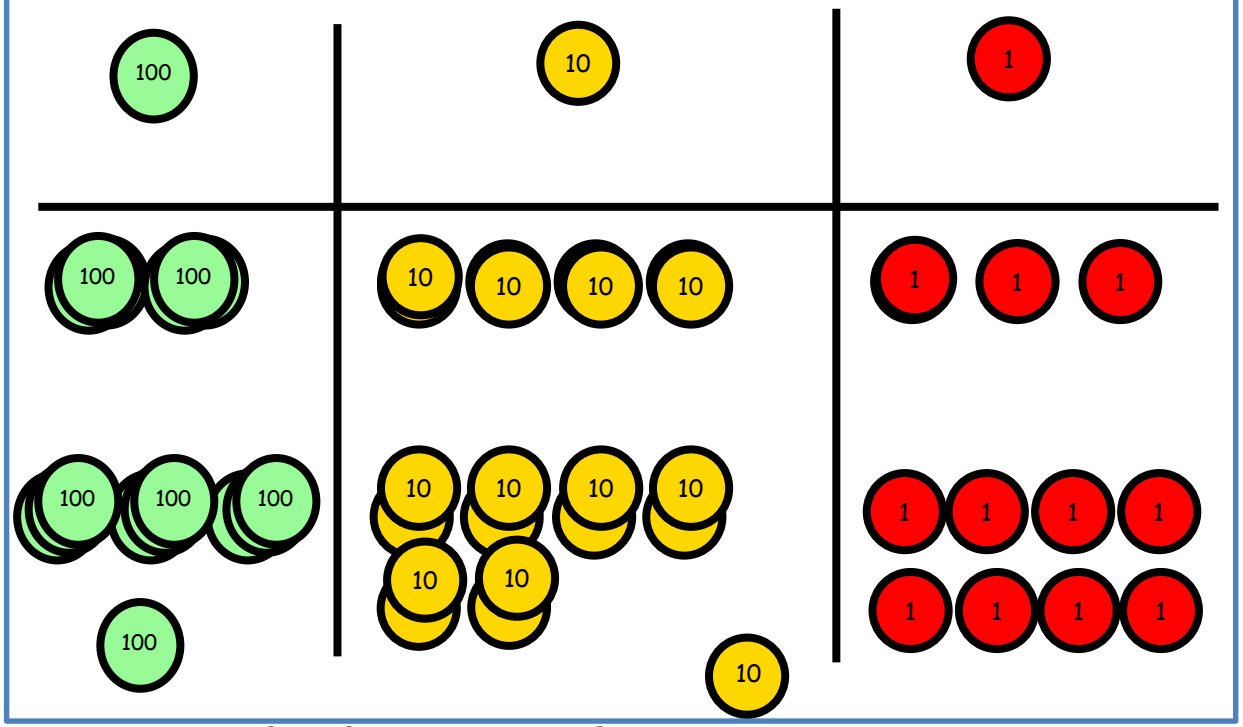
Key vocabulary: **exchange**

Can we exchange any counters?

Addition- Column method

Solve… 243 + 368 =

Model



Calculations

243

+368

1 1

6 11

Key vocabulary: **exchange**

Can we exchange any counters?

Moving to pictorial

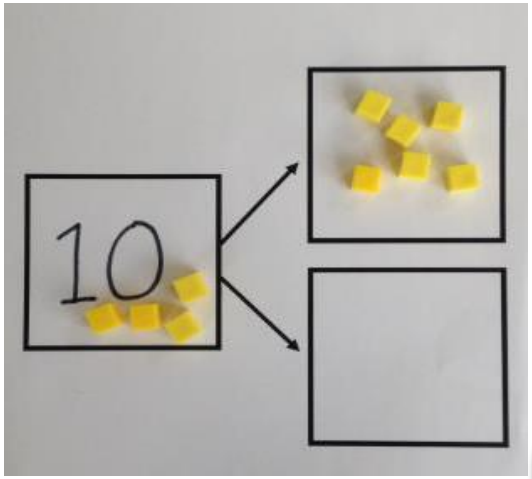
After lots of experience with the actual counters children should be able to draw the place value counters to help them solve a calculation, crossing out any counters that are to be exchanged.

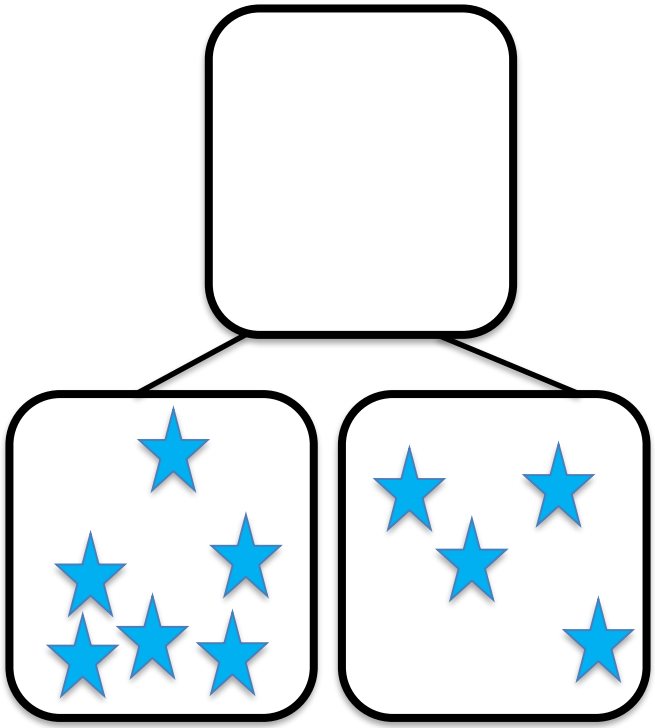
Subtraction

Subtraction- Part whole model

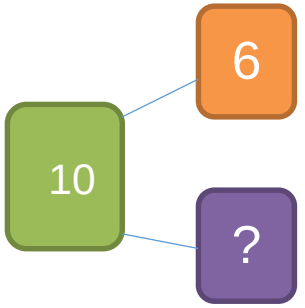
Solve… 10 – 6 =

Model





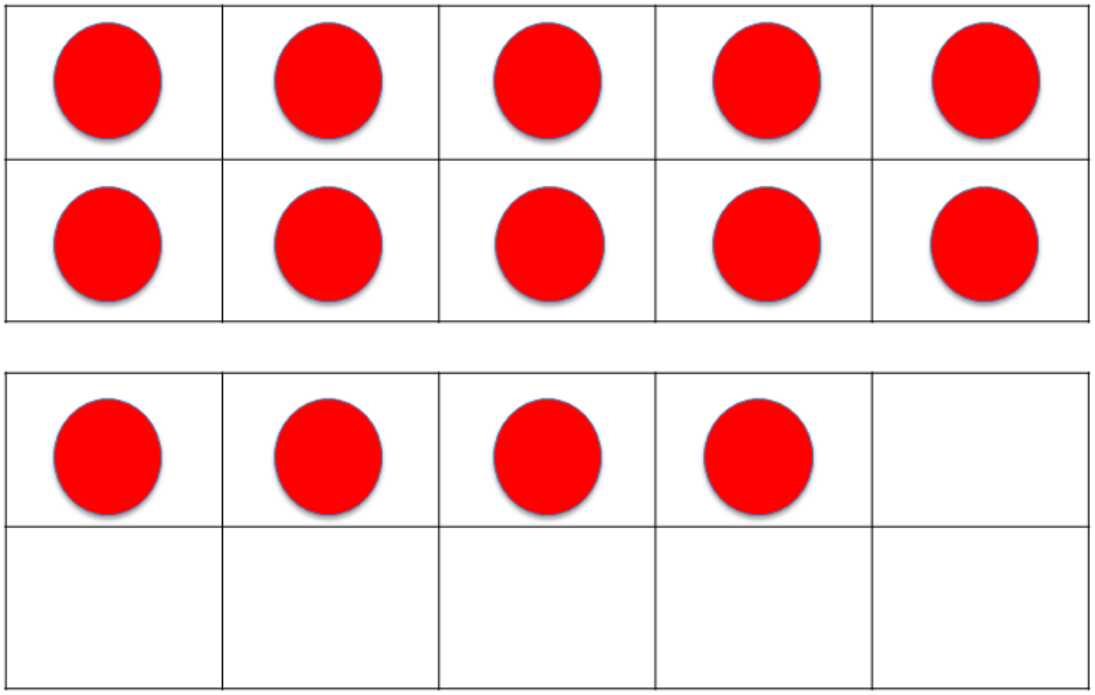
Calculations



Subtraction - make 10

Solve… 14 – 5 =

Model



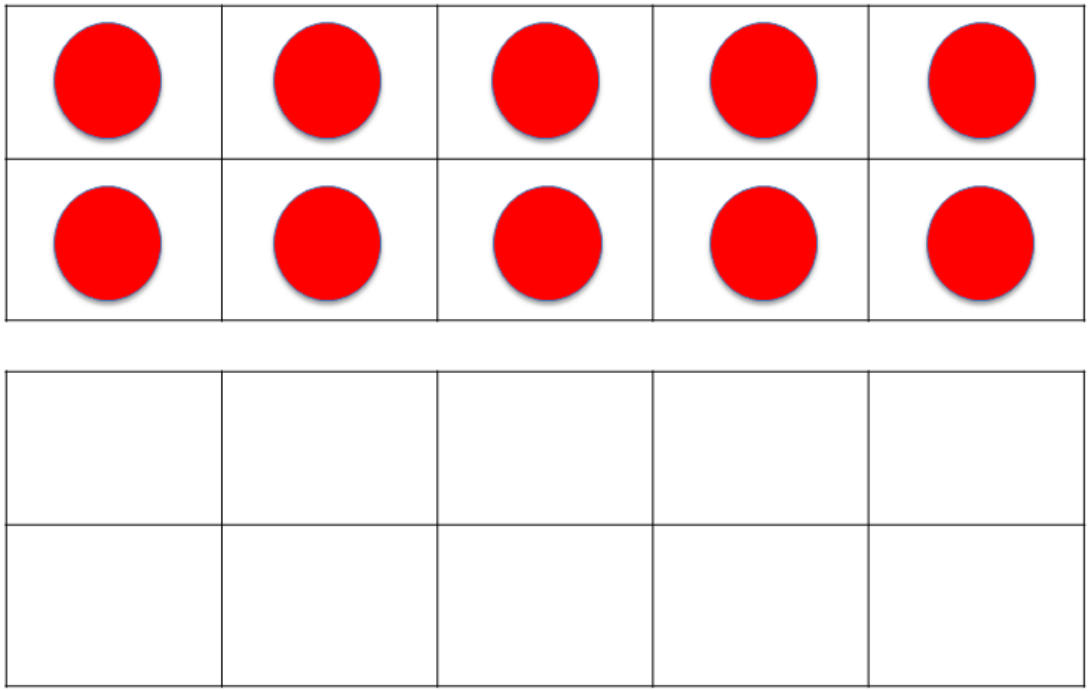
Calculations

14 – 5 =

Subtraction - make 10

Solve… 14 – 5 =

Model



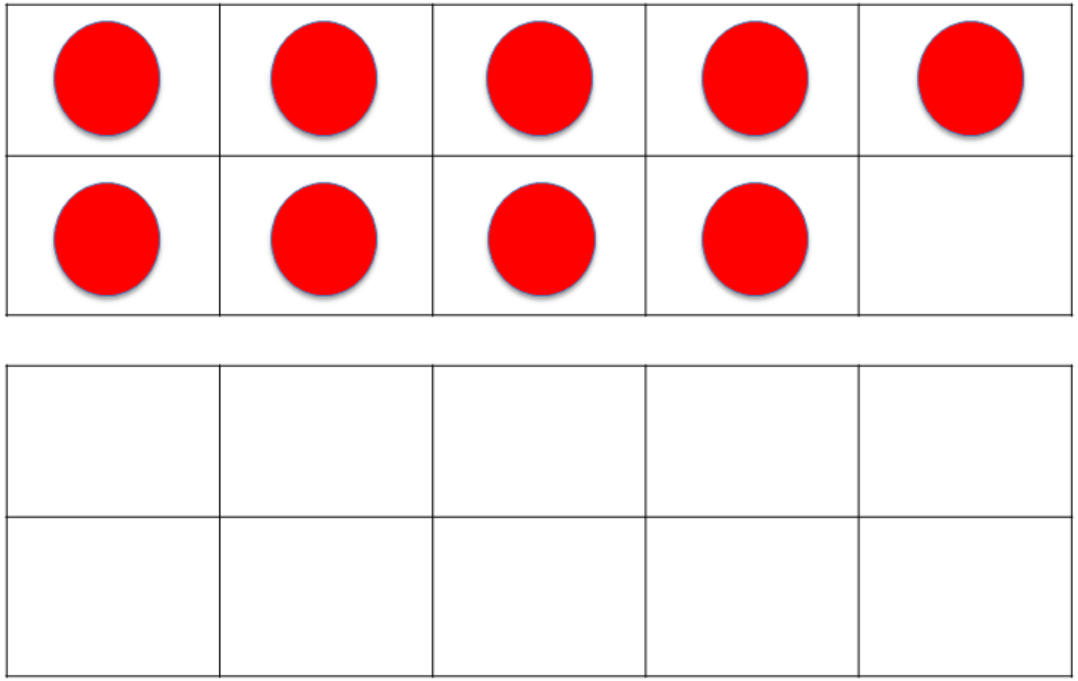
Calculations

14 – 5 =

Subtraction - make 10

Solve… 14 – 5 =

Model



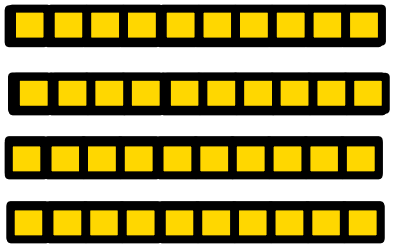
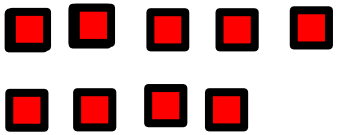
Calculations

14 – 5 = 9

Subtraction- column method

Solve… 49 – 27 =

Model

T	O
	

Calculations

49

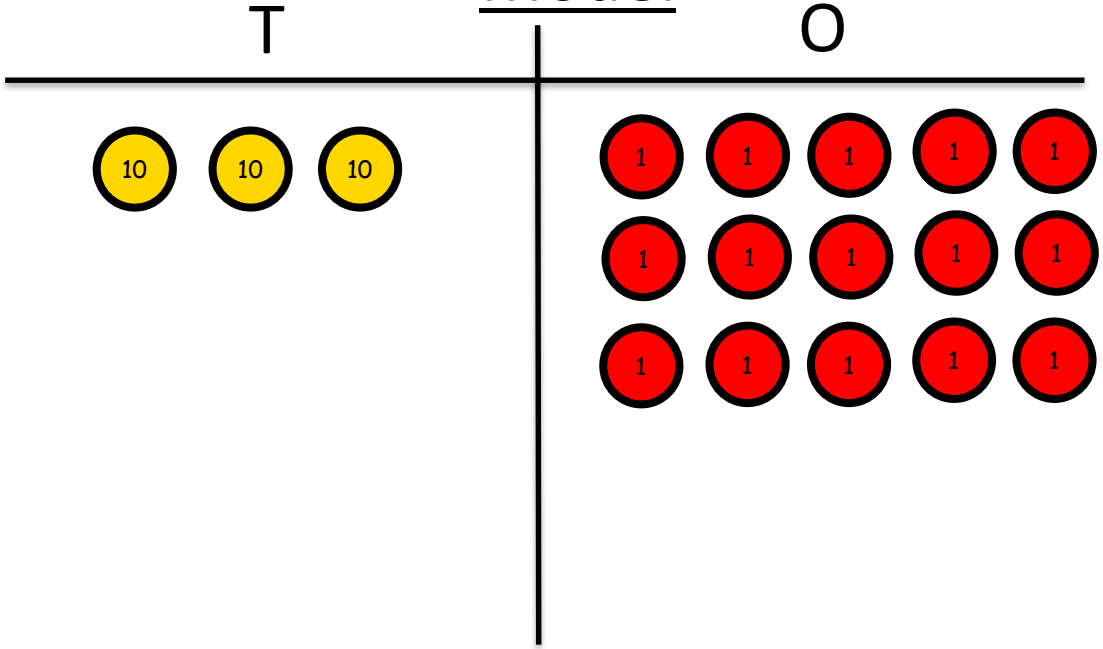
-27

22

Subtraction- Column method

Solve… 35 - 17 =

Model



Calculations

$$\begin{array}{r} {}^{2}\cancel{3}^{1}5 \\ -17 \\ \hline 18 \end{array}$$

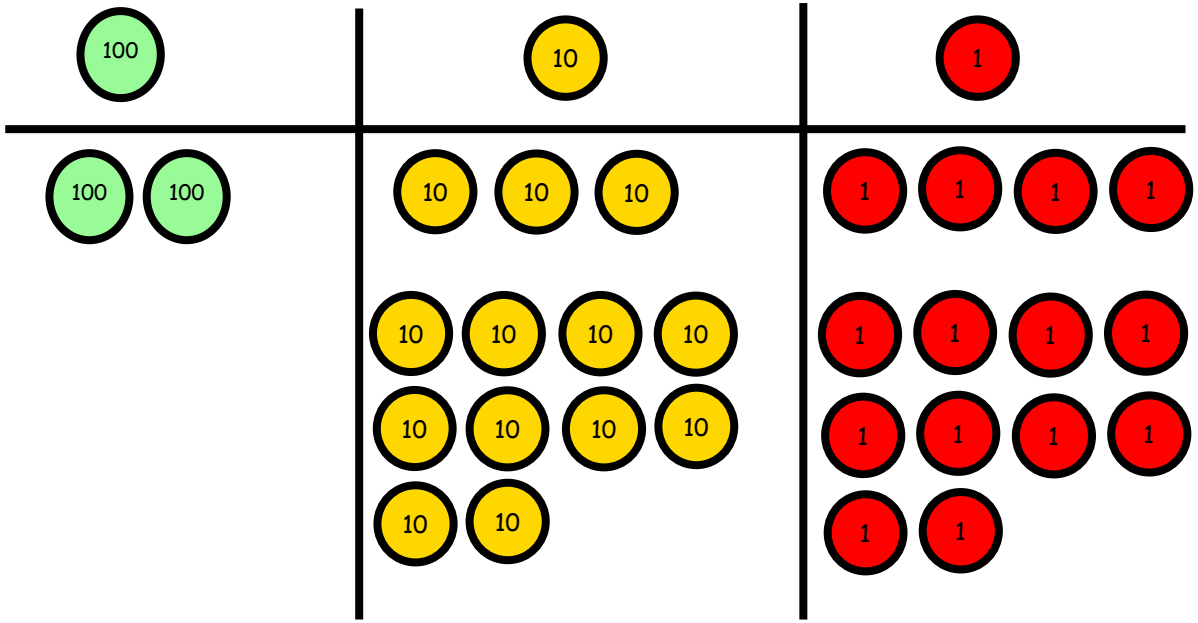
Key vocabulary: **exchange**

Can we exchange any counters?

Subtraction- column method

Solve… 234 – 88=

Model



Calculations

$$\begin{array}{r} {\scriptstyle 1\ \ 1\ \ 2\ \ 1} \\ 2\not{3}\not{4} \\ -\ \ 88 \\ \hline 146 \end{array}$$

Key vocabulary: **exchange**

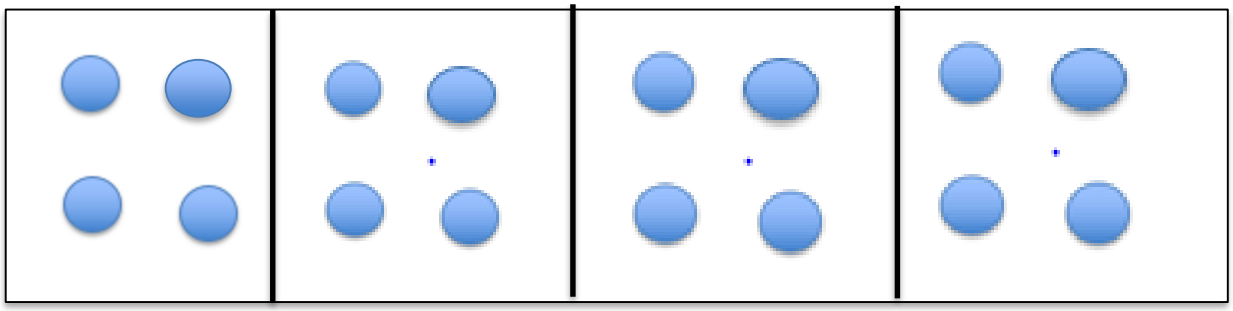
Can we exchange any counters?

Multiplication

Multiplication using concrete

16

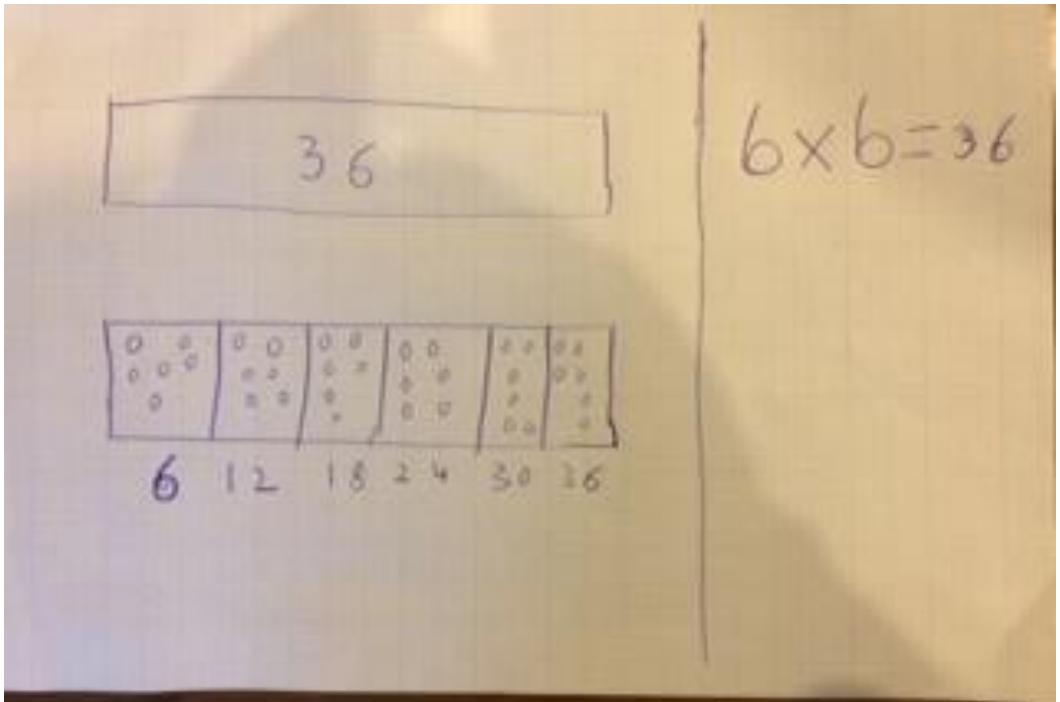
4 X 4 =



4 8 12 16

Pictorial with Place Value Counters

Children very quickly move on to using pictures to represent the place value counters and this can easily be done at home too.

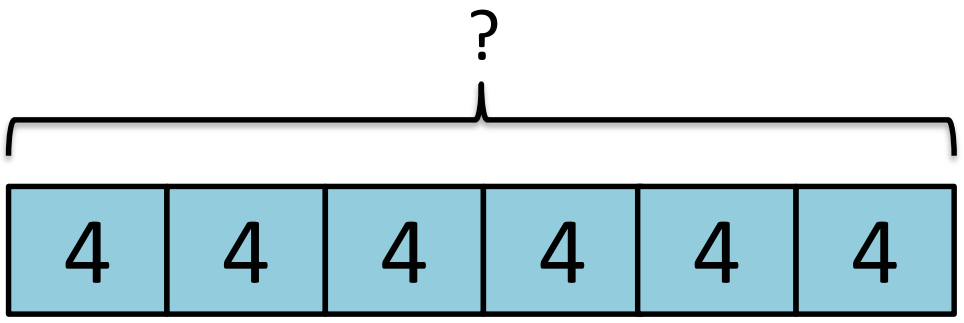


Multiplication

Muffins come in boxes of 4. Peter buys 6 boxes of muffins.

How many muffins does Peter buy all altogether?

Model

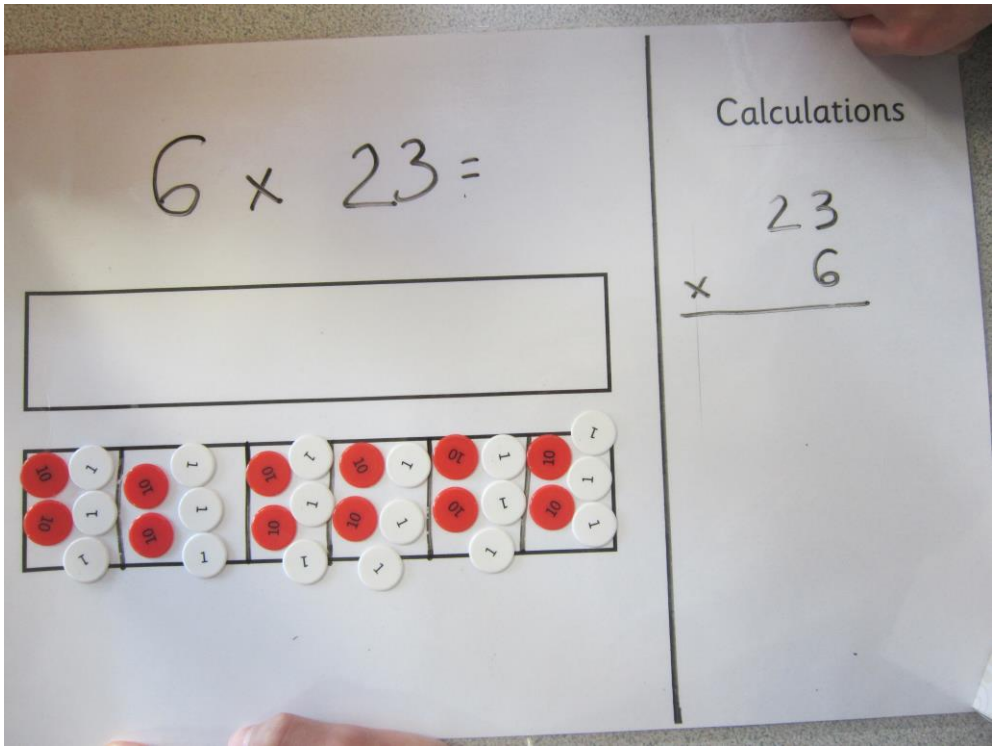


Calculations

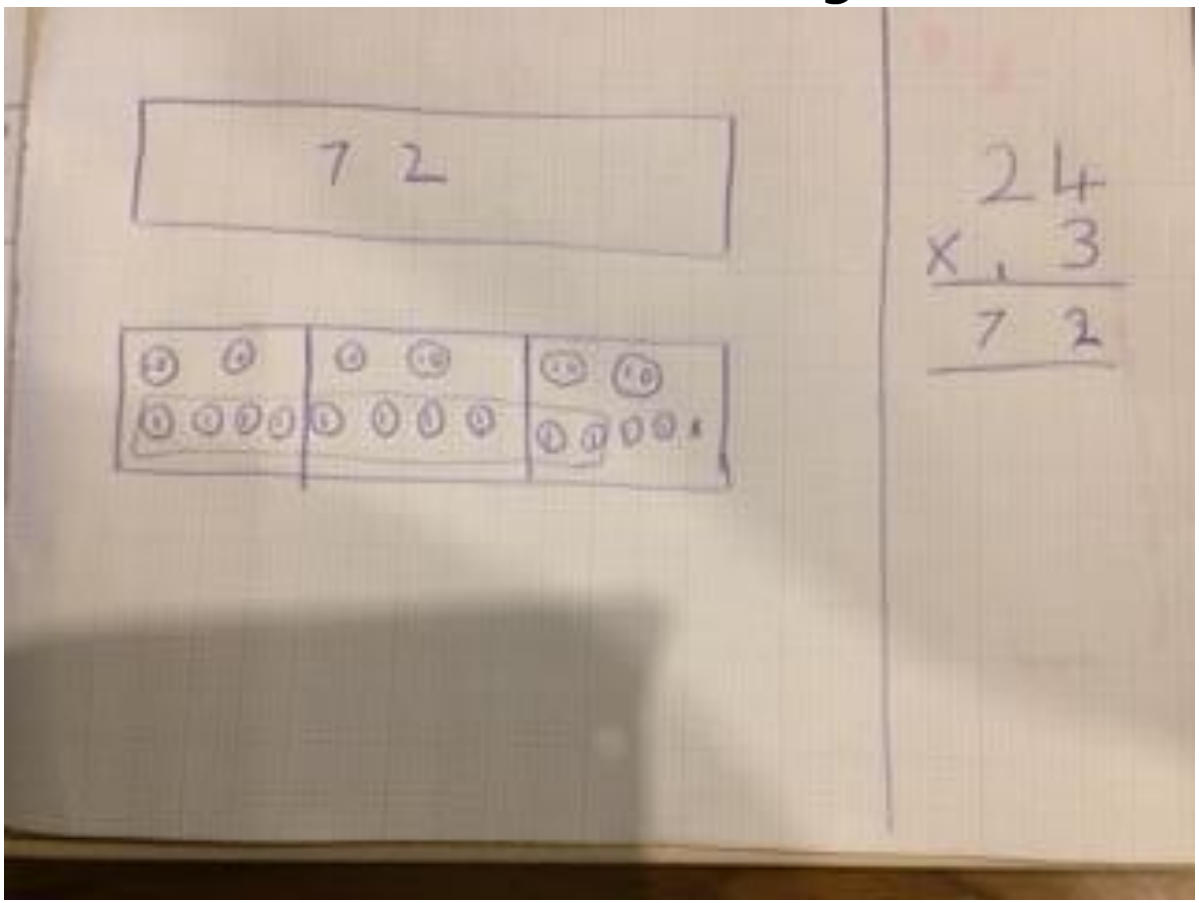
6 x 4=24

Moving to 2 digit numbers

Place value counters come into place again when we want to multiply 2 digit numbers.

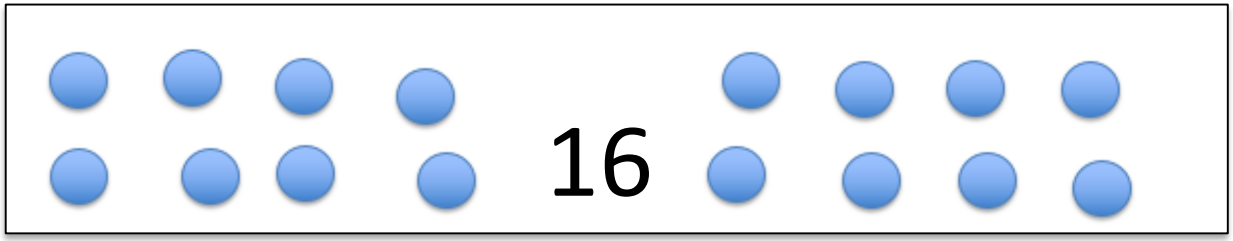


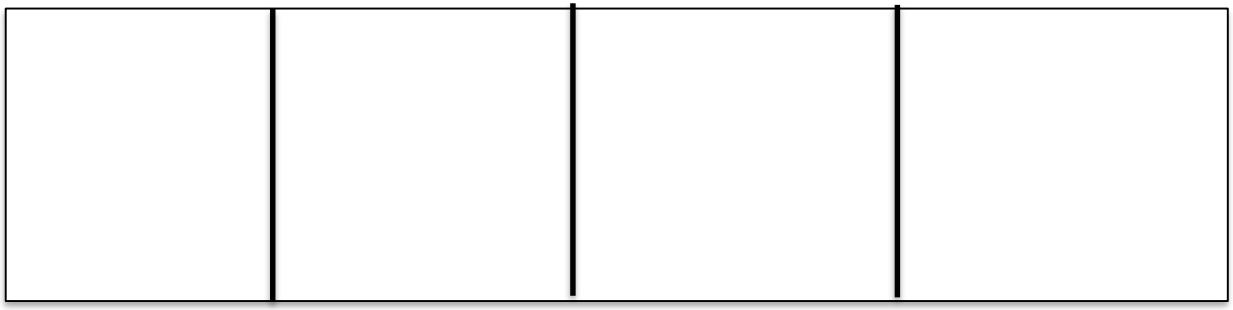
Pictorial with 2 digits



Division

Division using concrete

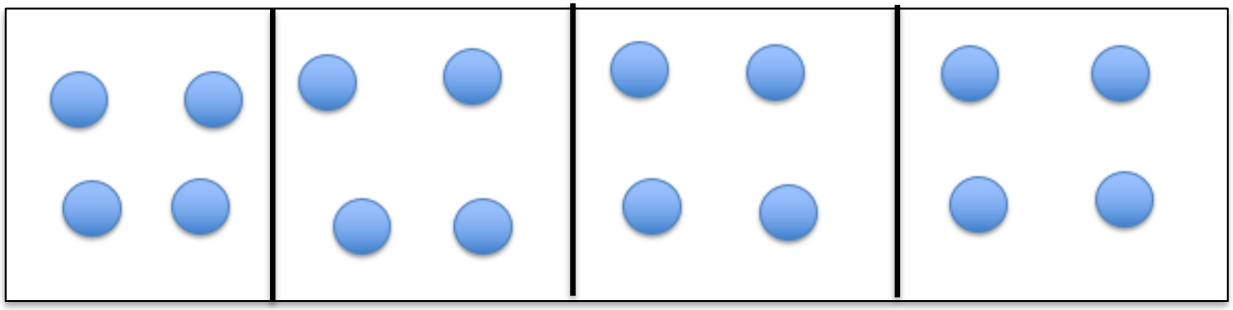




16 ÷ 4 =

Division using concrete

16

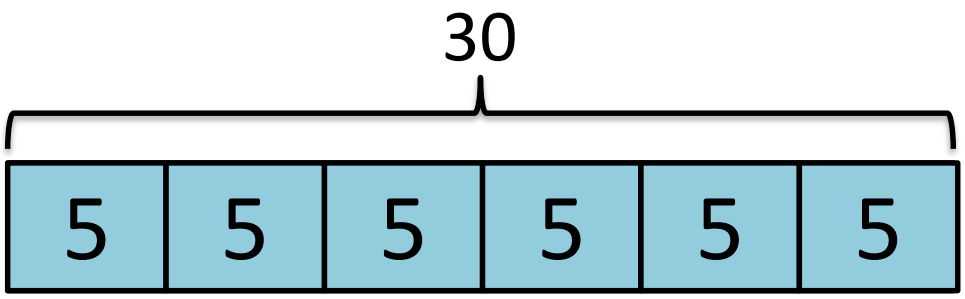


$16 \div 4 = 4$

Division (grouping)

Jane has 30 cakes. She wants to pack them into boxes with 5 cakes in each box. How many boxes will she need?

Model



Number of boxes needed = 6

Calculations

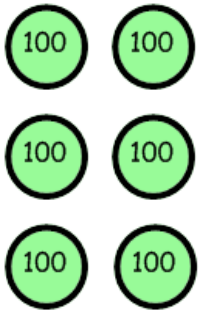
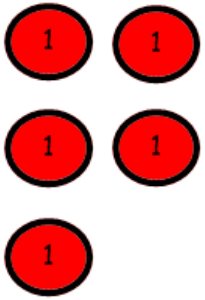
$30 \div 5 = 6$

In this version, we are counting how many fives go into thirty.

Division

Solve… 615 ÷ 5 =

Model

H	T	O
		

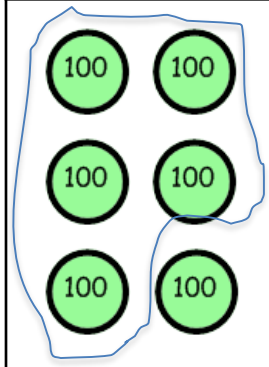
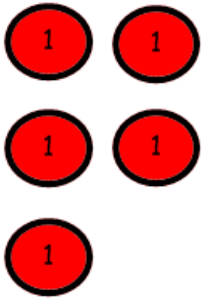
Calculations

$$5\overline{)615}$$

Division

Solve… 615 ÷ 5 =

Model

H	T	O
		

Calculations

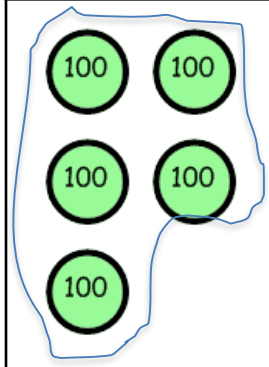
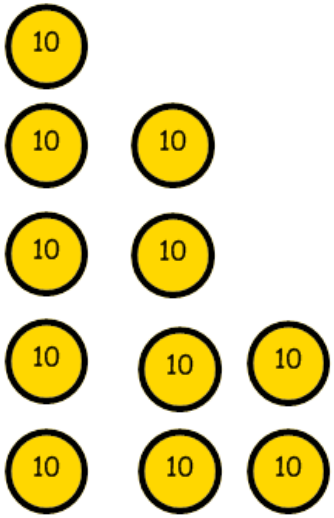
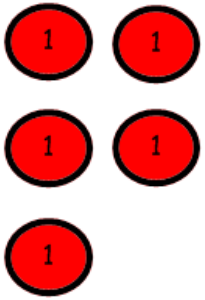
$$5\overline{)615}$$

1

Division

Solve… 615 ÷ 5 =

Model

H	T	O
		

Calculations

$$5\overline{)615}$$

1

Division

Solve… 615 ÷ 5 =

Model

H	T	O
100 100 100 100 100	10 10 10 10 10 10 10 10 10	1 1 1 1 1

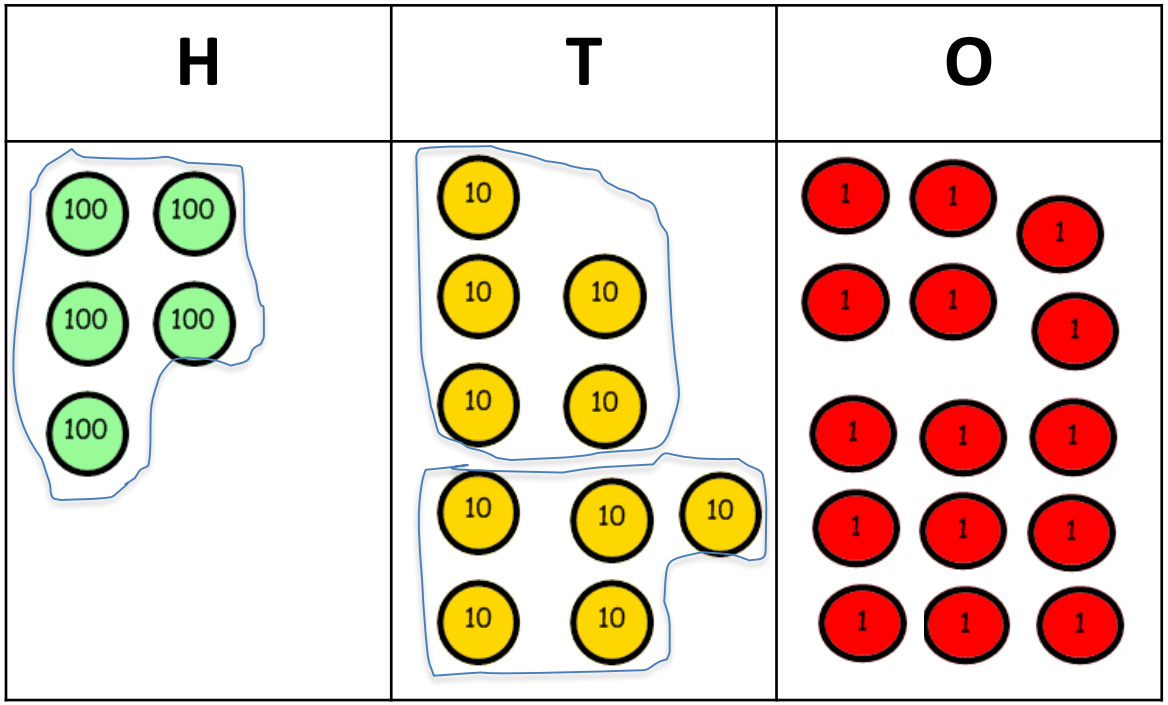
Calculations

$$5\overline{)615} = 12$$

Division

Solve… 615 ÷ 5 =

Model



Calculations

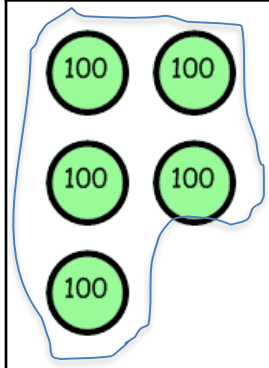
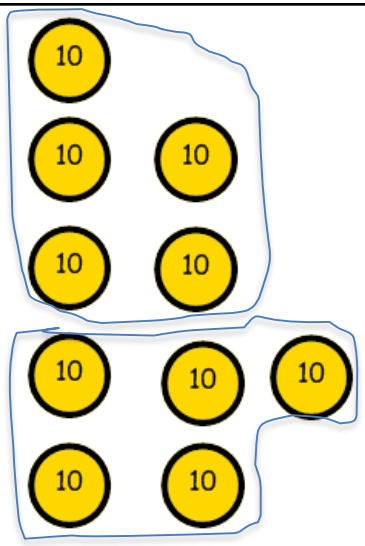
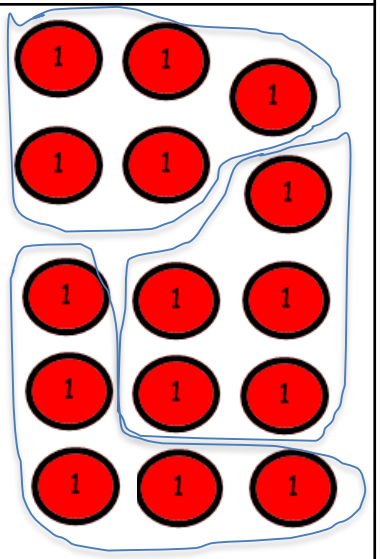
$$5\overline{)615}$$

$$12$$

Division

Solve… 615 ÷ 5 =

Model

H	T	O
		

Calculations

$$5\overline{)615}$$

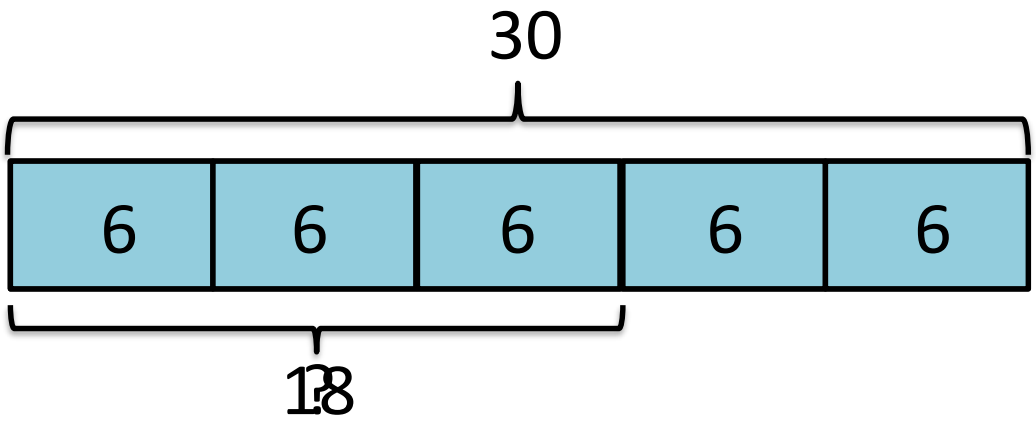
$$123$$

Fractions

Fraction of an Amount

Peter starts with 30 sweets. He eats $\frac{3}{5}$ of them. How many sweets does he eat?

Model



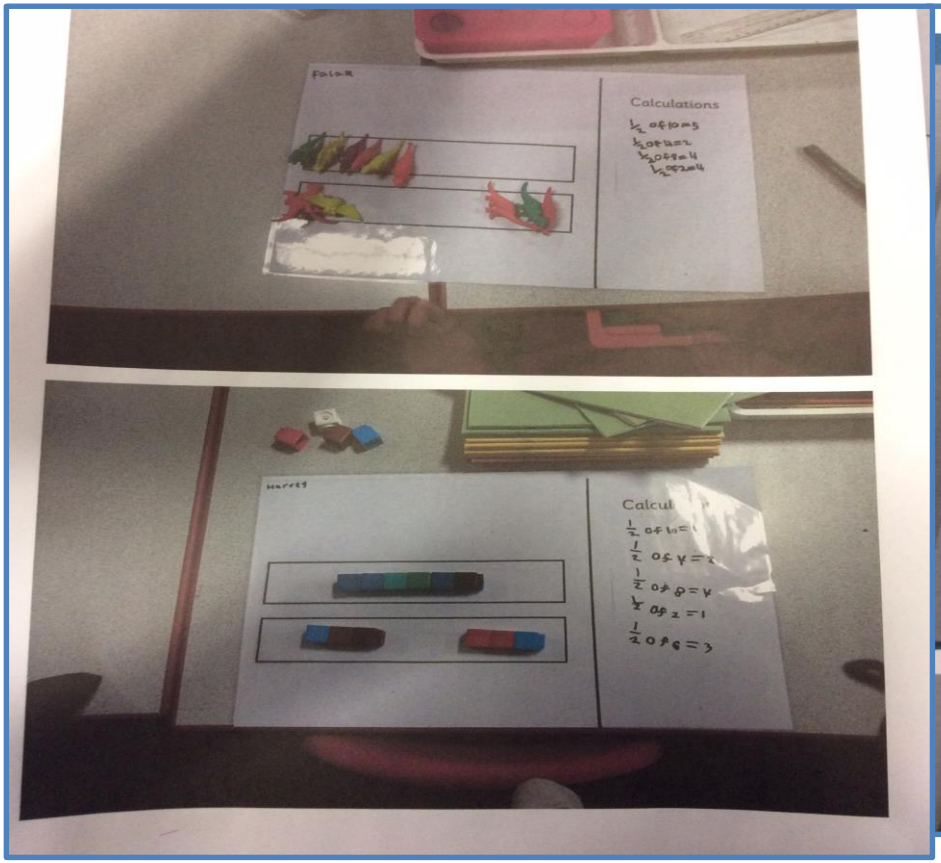
Calculations

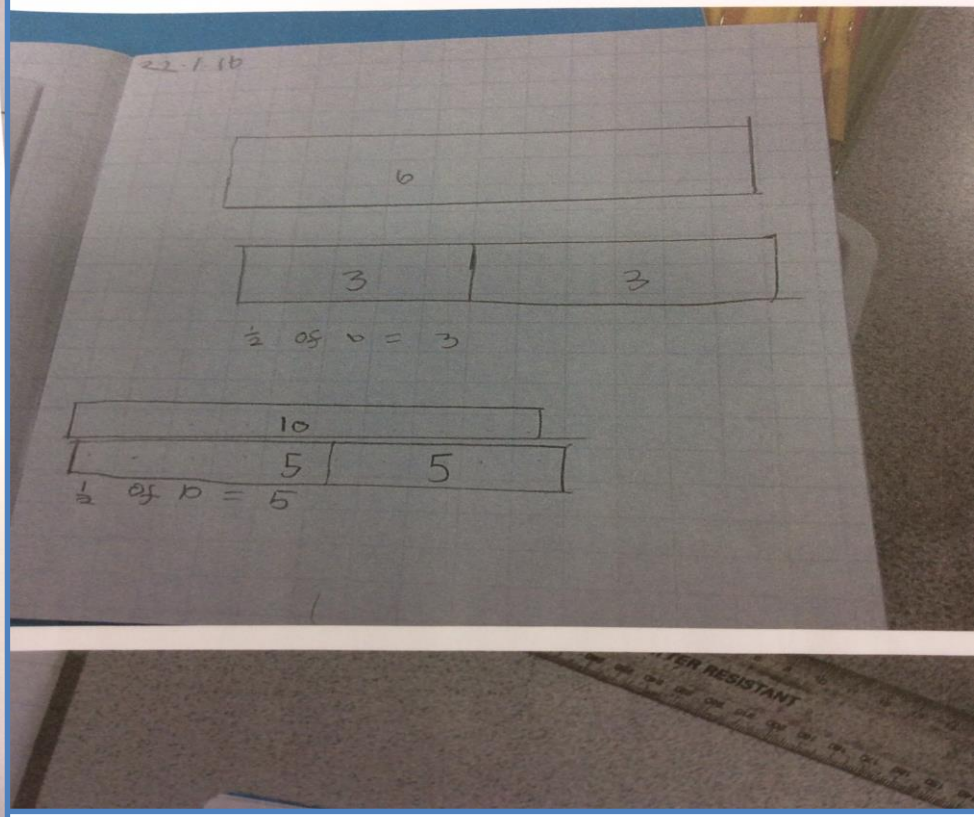
30 ÷ 5 = 6

6 x 3 = 18

Children's example

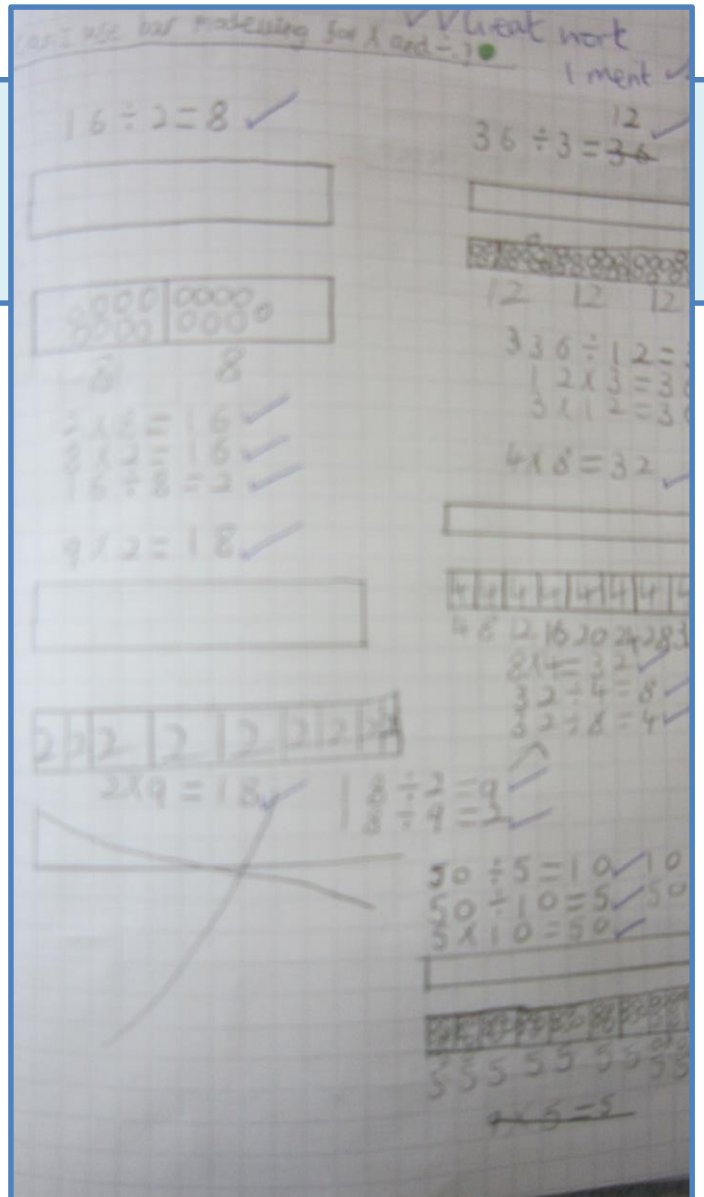
Year 1

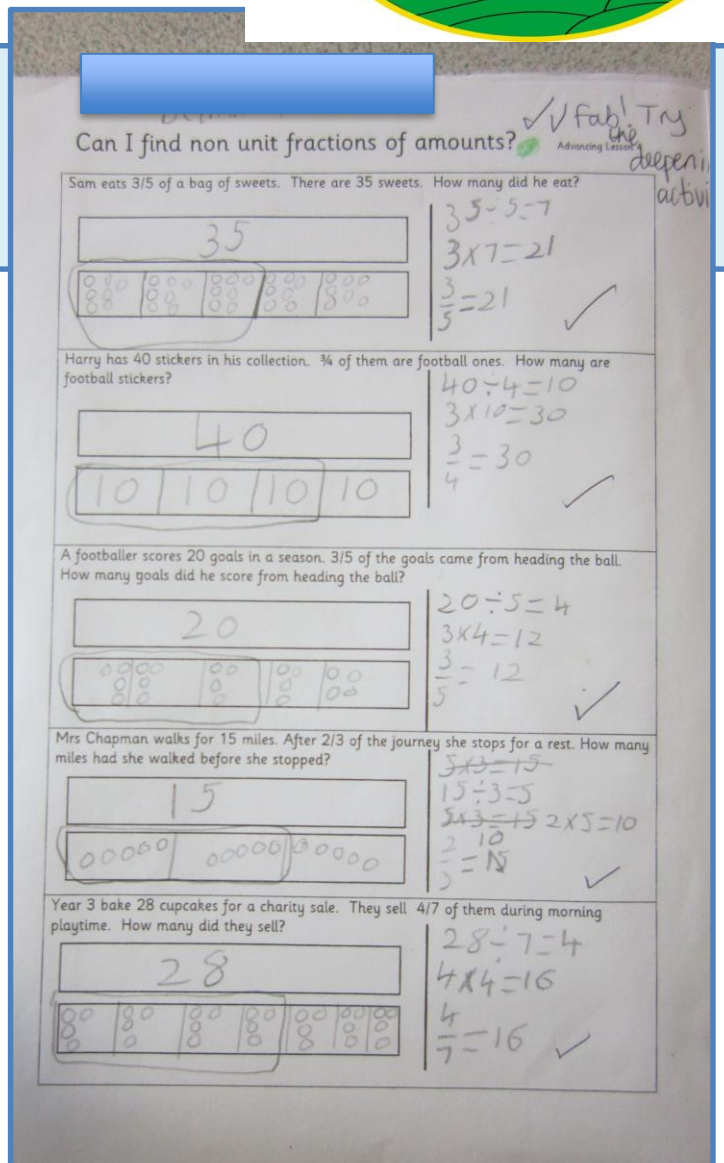




Children's example

Year 3

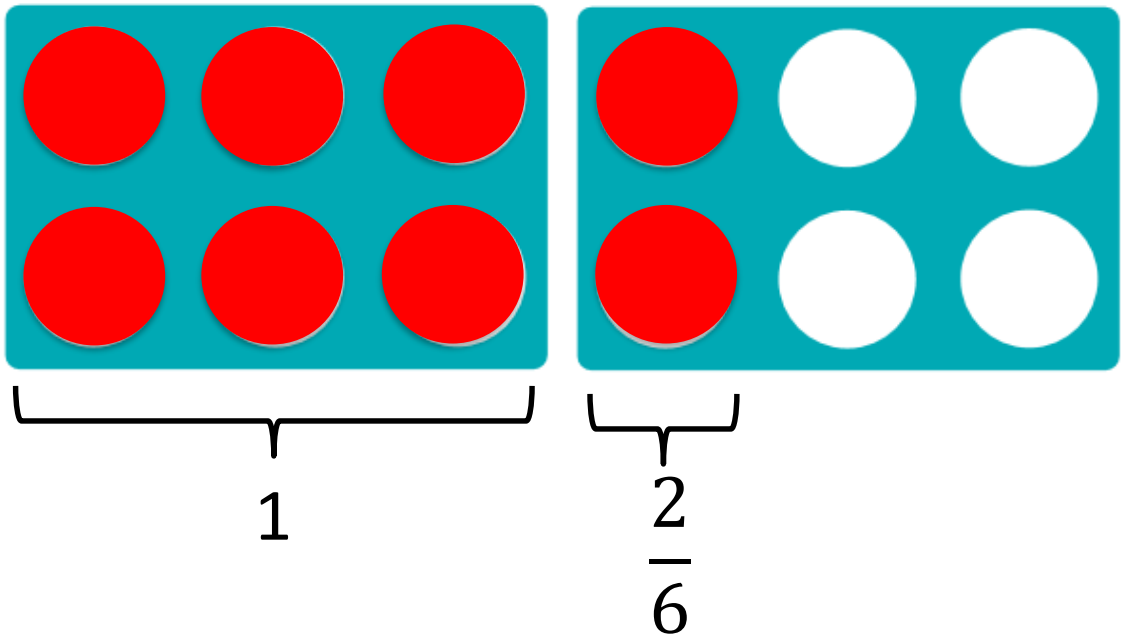




Converting fractions

Convert $\frac{8}{6}$ into a mixed number fraction

Model



Calculations

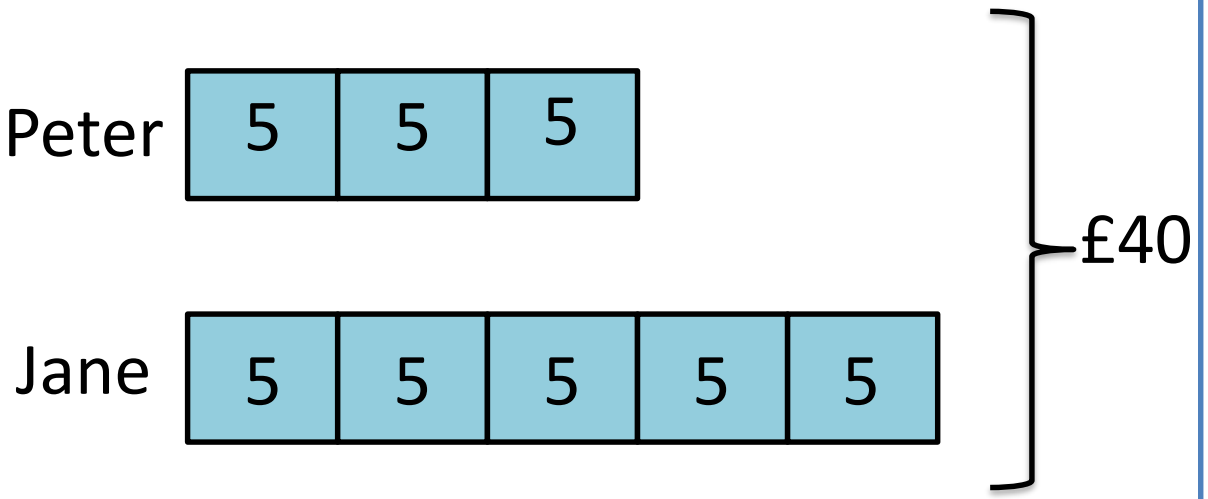
$$\frac{8}{6} = 1\frac{2}{6}$$

Ratio

Peter and Jane share £40 in the ratio of 3:5

How much money does each person get?

Model



Calculations

40 ÷ 8 = 5

Peter: 5x3 = 15

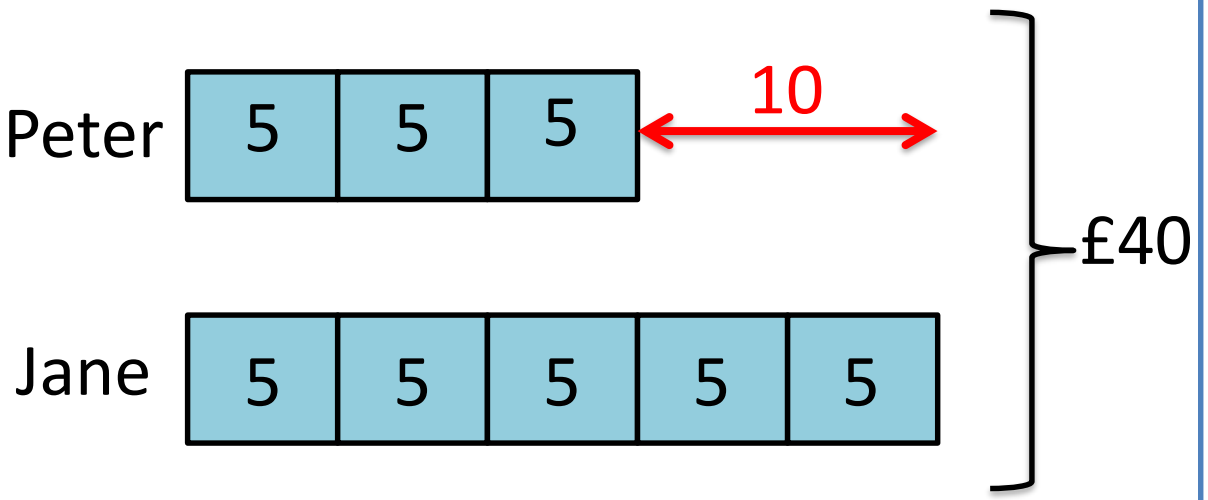
Jane: 5x5 = 25

Ratio

Peter and Jane share £40 in the ratio of 3:5

How much more money does Jane have than Peter?

Model



Calculations

40 ÷ 8 = 5

Peter: 5x3 = 15

Jane: 5x5 = 25

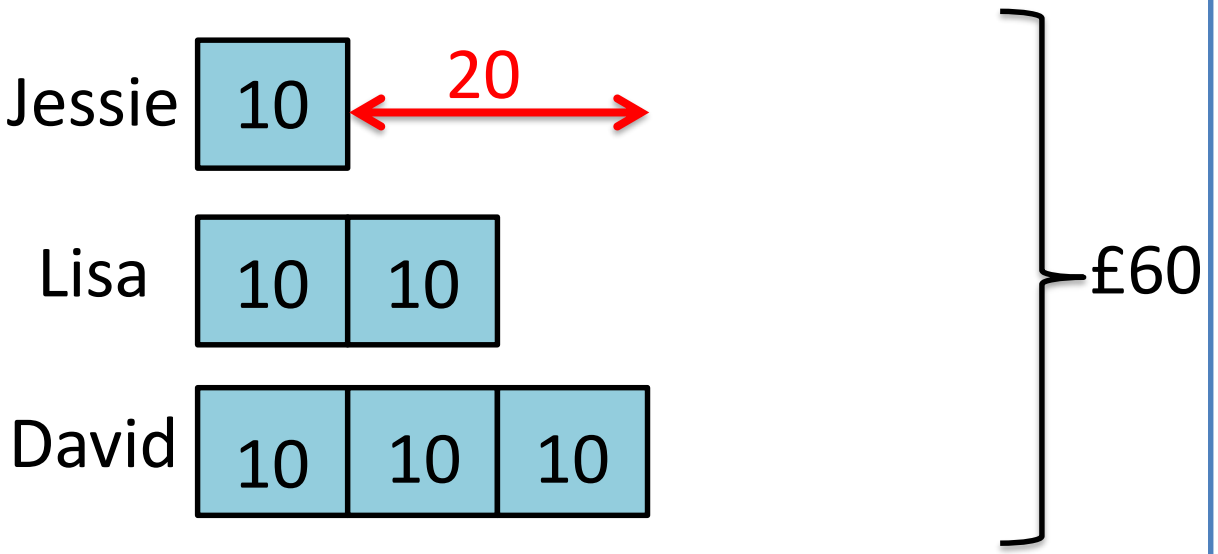
25 – 15 = 10

Ratio

Jessie, Lisa and David share £60 in the ratio of 1:2:3

How much more money does David get than Jessie?

Model



Calculations

60 ÷ 6 = 10

Jessie: 10x1 =10

Lisa: 10x2 = 20

David: 10x3 = 30

30 – 10 = 20

Thank you for listening.

We hope we have given you a useful insight into using CPA approaches within Mathematics.

If you have any questions please don't hesitate to ask.